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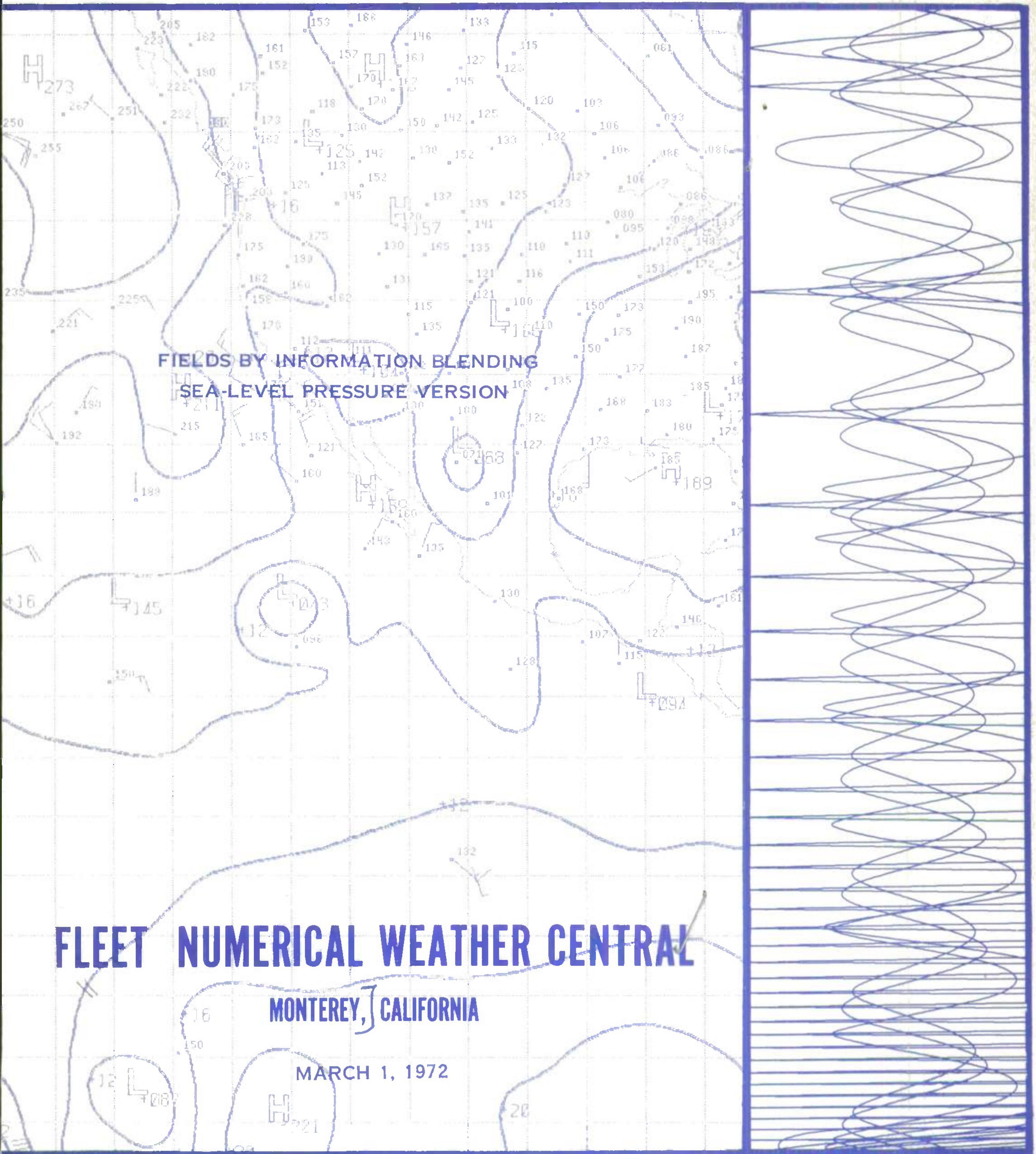

FIELDS BY INFORMATION BLENDING
SEA-LEVEL PRESSURE VERSION

FLEET NUMERICAL WEATHER CENTRAL

MONTEREY, CALIFORNIA

MARCH 1, 1972

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LEVEL II

THE FIB METHODOLOGY AND APPLICATION

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FIELDS BY INFORMATION BLENDING
SEA-LEVEL PRESSURE VERSION

by

Manfred M. Holl and Bruce R. Mendenhall

Prepared for

The Commanding Officer
Fleet Numerical Weather Central
Monterey, California

METEOROLOGY INTERNATIONAL
INCORPORATED

MONTEREY, CALIFORNIA

Meteorology International Incorporated
Monterey, California

Project M-167
Final Report
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FIB

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Sea-Level Pressure Version

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FIB STATUS REPORT

The attached report* constitutes the most detailed description that we have produced to date of our

Fields by Information Blending (FIB) Methodology.

We believe this new, comprehensive approach to be the most advanced and definitive available for the objective analysis of distributions of specific types of object parameters. The methodology is applicable to any type of object parameter, scalar or vector, having significant character expressible by linear spatial operators acting on the object distribution (e.g., information as to gradient components, Laplacian and other higher derivatives, divergence, curl, etc.).

The methodology is based on the treatment of information as a metered commodity: Every input piece of information is weighted as to its purported independent worth. This reliability weight is defined as the inverse of the variance associated with the piece of information. Information in direct observations (i.e., spot measurements) of the object parameter, and information, observed and/or constructed, as to spatial characteristics of the object distribution, are assembled by reliability weights and blended to give the best resultant distribution of the object value, and the associated distribution of the remaining, unresolved variance.

The attached report deals with the application of the methodology in the development of an objective analysis scheme for Sea-Level-Pressure (SLP) distribution. References are made to the simpler application of the

*The attached report has also been reproduced under the covers of the Fleet Numerical Weather Central, as Technical Note No. 72-2.

methodology for Sea-Surface-Temperature (SST) analysis*, and to the more complex application for horizontal wind (UV) analysis.

At this time, the following applications are in operational use by the Fleet Numerical Weather Central:

FIB/SST Northern Hemisphere Oceans, 125x125 grid array (half mesh length)

FIB/SST Gulf Stream Region, 63x63 grid array (quarter mesh length)

FIB/SLP Northern Hemisphere, 125x125 grid array (half mesh length)

The mesh length refers to the grid spacing of the standard FNWC 63x63 grid array of the polar stereographic projection encompassing the northern or southern inscribed hemisphere.

The following applications are currently under development:

FIB/SST Southern Hemisphere Oceans, 63x63 grid array (standard mesh length)

FIB/OTS Analysis of Ocean Thermal Structure (OTS) Parameter Distributions

FIB/SLP Med. Greater Mediterranean Region, 63x63 grid array (quarter mesh length)

The following applications have been formulated:

FIB/UV Horizontal Wind Analysis

FIB/UA Upper Air Analysis, isobaric height and horizontal wind structure parameters

Other applications are under formulation.

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*Holl, Manfred M., Bruce R. Mendenhall and Charles E. Tilden, 1971; "Technical Developments for Operational Sea Surface Temperature Analysis with Capability for Satellite Data Input", Final Report, Contract No. N62306-70-C-0334, (Naval Weapons Engineering Support Activity), Meteorology International Incorporated, Monterey, California.

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1. Introduction

A comprehensive technique for the objective analysis of scalar and of vector fields has been developed by Meteorology International Incorporated, under Navy contracts. This technique has been designated the Fields by Information Blending (FIB) technique.

Information is a delicate commodity which is easily depreciated by transformation and assimilation schemes. The FIB technique treats information as a metered commodity. It assembles all available information, which must include suitable weighting, and blends the information into resultant analyses and associated distributions of resultant resolution weight.

In the FIB context, all information statements must include both parameter estimates and associated reliabilities. Any statement of information, whether observation or analysis value at a grid point, is incomplete without an associated reliability.

For an independent piece of information the reliability, or report weight, is defined as the inverse of the error variance inherent in the observation and/or associated with the class of observation. For a statement of resolution the associated reliability, or resolution weight, is the inverse of the unresolved variance.

The FIB technique is based entirely on rules for adding uncorrelated variance contributions and for adding independent information. The operation which produces the implied resultant, given weighted grid-point arrays of values of the object parameter and of its linear integral and finite-difference derivatives, is called the blending.

The information available for producing an analysis may be considered to be from three sources in time:

(1) Concurrent observations. Observations taken at or very near the applicable time of the object analysis.

(2) Near-past observations. This information has been assimilated into the analysis preceding, and is projected forward along the time axis in the form of first-guess fields.

(3) Well-past observations. This is the source of physical and statistical relationships. A pertinent example is the geostrophic approximation by which wind information is transformed into pressure gradient information and vice versa. Note that the transformation contributes an error variance to the information so transformed. Well-past observations are also the source of specific inferences such as those grouped under area synoptic skills of an analyst.

The information available also differs as to type and relevance. The FIB technique can directly assimilate information in observations (i.e., random samplings) of the parameter and information as to integral and differential field properties.

Two versions of the FIB technique have been accomplished to date:

FIB/SST: Application to Sea-Surface Temperature with a hemispheric (125x125 grid array) and a fine mesh (63x63) regional adaptation.

FIB/SLP: Application to Sea-Level Pressure with a hemispheric (125x125) adaptation.

A third version has been formulated:

FIB/UV: Application to a horizontal-velocity field (i.e., wind analysis).

FIB/SST directly assimilates information in the form of (1) temperature values, and (2) temperature-gradient values in two components. The temperature gradient is expressed in terms of temperature difference per grid interval. It has two components, associated with the two grid-array coordinate axes.

FIB/SLP directly assimilates information in the form of (1) pressure values, (2) pressure-gradient values in two components, such as derived from winds, and (3) Laplacian values for pressure, expressed in terms of a five-point finite-difference operator, in the grid scale.

FIB/UV directly assimilates information in the form of (1) wind values in two components, (2) vorticity in the grid scale, and (3) divergence in the grid scale.

Figures 1, 2 and 3 relate to FIB/SST, FIB/SLP and FIB/UV respectively. Each indicates the reference locations for parameter values in an arbitrary area module of a rectangular grid array.

The FIB technique includes the capability to accept artificial data whether generated objectively or subjectively by interpretation of additional information such as from satellites or by preferred interpretation of surface data. The artificial data must be expressed in the direct forms and must include estimated reliability weights.

In the blending operation the spreading of observations (i.e., individual independent reports) is effected by the highest-order fields--that is, by the resolution fields which express information derived from higher-order differential properties of the field which control shape and character:

For FIB/SST this is the gradient resolution in two components.

For FIB/SLP this is the Laplacian resolution field.

For FIB/UV this is expressed by the vorticity resolution field and the divergence resolution field.

These are the basic spreaders. The blending of observations, and the exploitation of first-guess information, can be enhanced by expressing information in additional differential properties of the fields.

The FIB technique, in general, includes the following component operations:

(1) Preparation of first-guess fields from available and generated sources.

(2) Assembly of new information. Independent estimates in any of the acceptable forms are assembled by weighted combination at grid-point reference locations, but the highest-order fields must be restructured to accept new information.

(3) Blending of information fields into resultant analysis.

(4) Evaluation of reliability weights, performed by limited reanalysis using a perturbed value at each grid point.

(5) Gross-error checking and reevaluations. Includes capability for monitoring quality of station reports.

(6) Recycling of Assembly, Blending and Resultant-Weight operations.

These operations are detailed for FIB/SST in a technical report*. The operations for FIB/SLP are detailed in the present report.

A major source of information for sea-level pressure analysis is represented by surface wind observations. FIB/SLP exploits much of this information by transforming each wind into the two pressure-gradient components, according to balanced large-scale dynamics, neglecting local wind tendency. Those surface winds which appear to be non-representative of the large-scale dynamics may be rejected in the gross-error check operation. In the synoptic scale this treatment exploits most of the relevant information inherent in the winds.

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*Holl, Manfred M., Bruce R. Mendenhall and Charles E. Tilden, 1971; "Technical developments for operational sea surface temperature analysis with capability for satellite data input", Final Report, Contract No. N62306-70-C-0334 (Naval Weapons Engineering Support Activity), Meteorology International Incorporated, Monterey, California, 46pp.

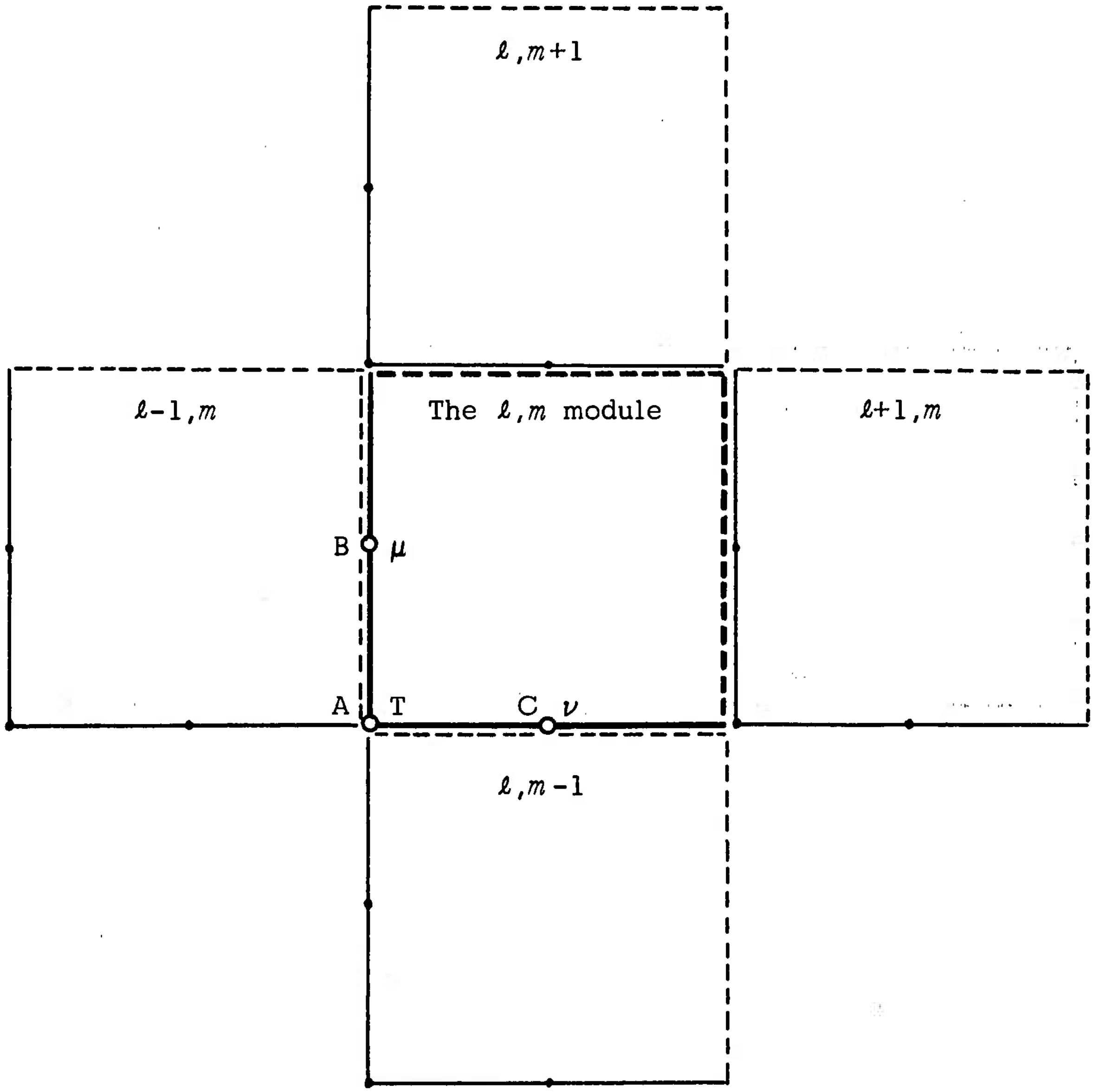

Fig. 1 The arbitrary ℓ,m area module (consisting of one corner point, two sides, and the interior area) and adjoining modules. Reference locations in the module are shown for the Sea-Surface-Temperature parameter, $T_{\ell,m}$, and the finite-difference parameters:

$$\mu_{\ell,m} \Rightarrow T_{\ell,m+1} - T_{\ell,m} \quad \text{and} \quad \nu_{\ell,m} \Rightarrow T_{\ell+1,m} - T_{\ell,m}$$

The respective reliabilities of T , μ and ν values are denoted by A, B and C; the reliability of a value is defined as the inverse of the error variance associated with the value.

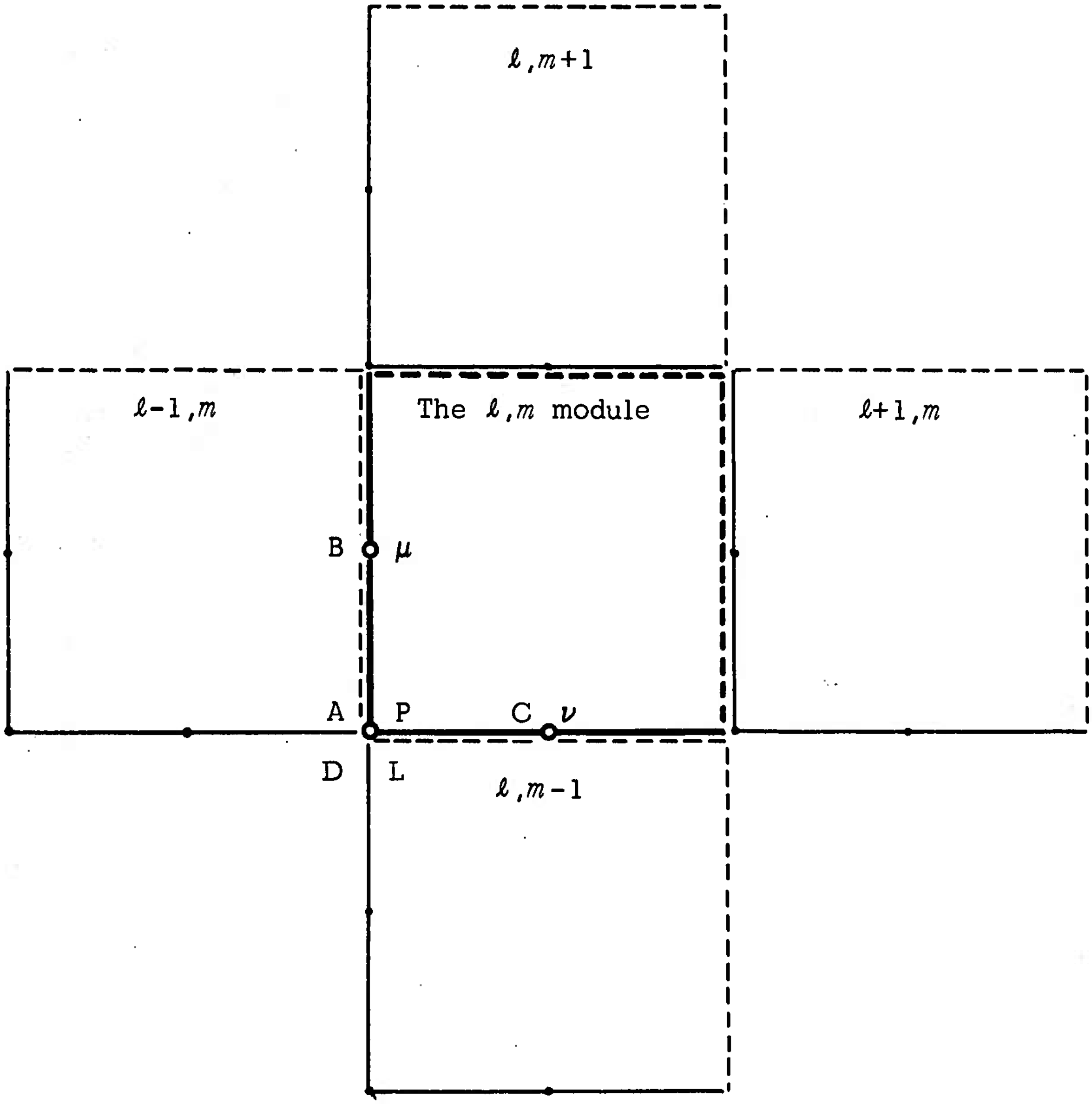

Fig. 2 The arbitrary ℓ,m area module (consisting of one corner point, two sides, and the interior area) and adjoining modules. Reference locations in the module are shown for the Sea-Level-Pressure parameter, $P_{\ell,m}$, and the finite-difference parameters:

$$\mu_{\ell,m} \Rightarrow P_{\ell,m+1} - P_{\ell,m} \; ; \; \nu_{\ell,m} \Rightarrow P_{\ell+1,m} - P_{\ell,m} \; ;$$

$$\text{and} \quad L_{\ell,m} \Rightarrow P_{\ell+1,m} + P_{\ell,m+1} + P_{\ell-1,m} + P_{\ell,m-1} - 4\,P_{\ell,m}$$

The respective reliabilities of P, μ, ν, and L values are denoted by A, B, C and D; the reliability of a value is defined as the inverse of the error variance associated with the value.

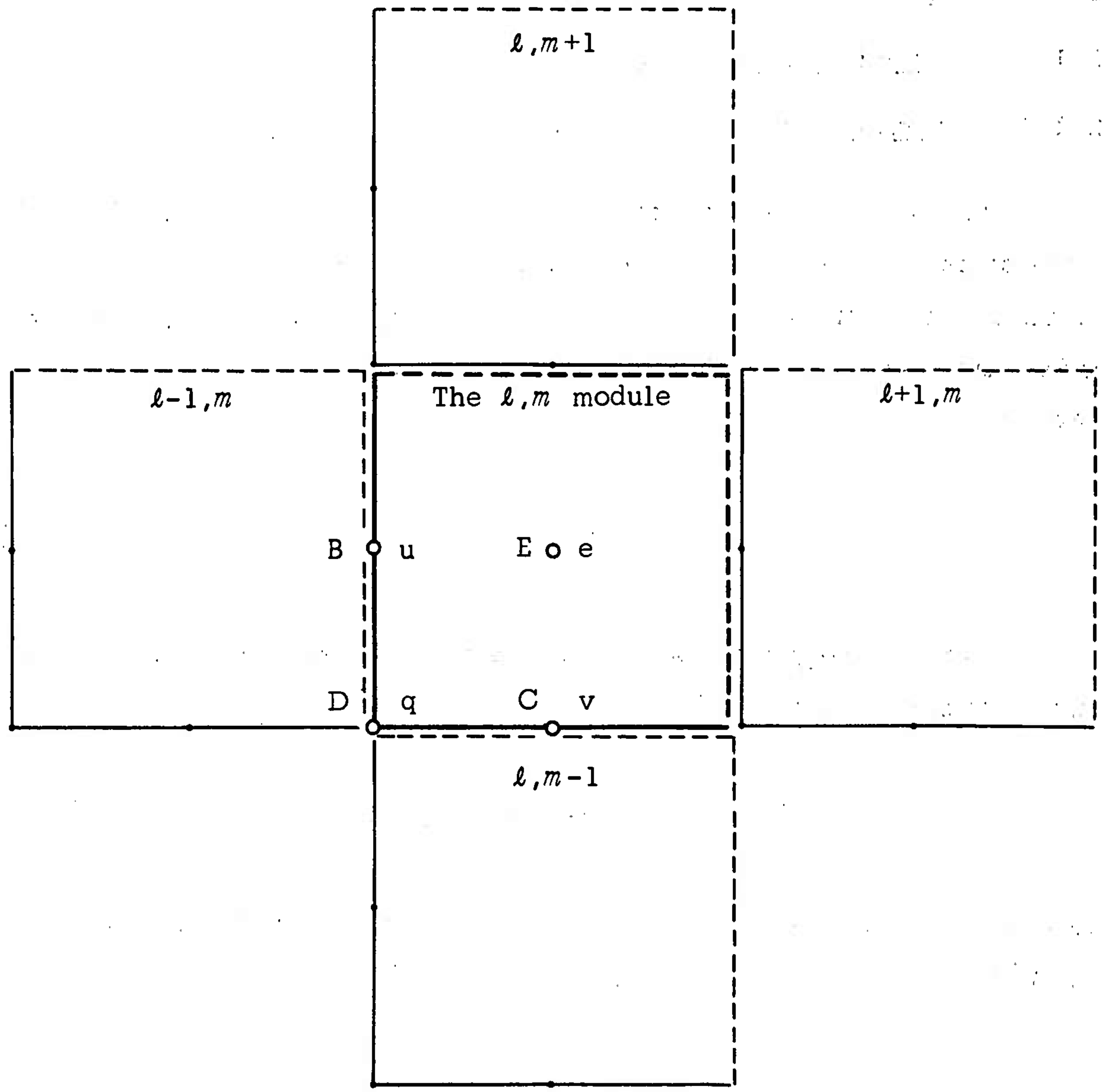

Fig. 3 The arbitrary ℓ,m area module (consisting of one corner point, two sides, and the interior area) and adjoining modules. Reference locations in the module are shown for the u and v components of the wind velocity, and the finite-difference parameters:

$$e_{\ell,m} \Rightarrow u_{\ell+1,m} - u_{\ell,m} + v_{\ell,m+1} - v_{\ell,m} ;$$

and $$q_{\ell,m} \Rightarrow v_{\ell,m} - v_{\ell-1,m} - u_{\ell,m} + u_{\ell,m-1} .$$

The respective reliabilities of u, v, q and e values are denoted by B, C, D and E; the reliability of a value is defined as the inverse of the error variance associated with the value.

2. Conceptions and Basic Formulas

2.1 Definitions and Rules

2.1.1 Definitions

Let p_n denote an independent estimate (e.g., measurement) of sea-level pressure. Let σ_n denote the standard deviation associated with this estimate. The variance is given by σ_n^2. The weight or reliability of the information expressed by the estimate is defined as the inverse of the variance:

$$A_n \equiv 1/\sigma_n^2 \quad . \tag{1}$$

Let μ_m denote an estimate of the finite difference in pressure between two locations, m and $m + 1$:

$$\mu_m \Rightarrow \text{an estimate of } p_{m+1} - p_m \quad . \tag{2}$$

The weight of μ_m is denoted by B_m. The associated variance is then given by B_m^{-1}.

2.1.2 Addition of Contributing Variances

Estimates are independent if the associated errors are mutually uncorrelated. For a linear combination of such independent estimates,

$$p = p_o \pm \mu_1 \pm \mu_2 \pm \ . \ . \ . \quad , \tag{3}$$

the associated variance is given by

$$\sigma^2 \equiv A^{-1} = A_o^{-1} + B_1^{-1} + B_2^{-1} + \ldots \quad . \tag{4}$$

For the product of an estimate, p, multiplied by a constant, c, the variance is $c^2\sigma^2$.

2.1.3 Relevant Variance in a Measurement

With respect to the analysis of a spatial distribution, the contributing variances in a measure, relative to an objective resolution as to scale, are

$$\sigma^2 = \sigma^2 \text{ (instrument system)} + \sigma^2 \text{ (subscale noise)} \quad . \tag{5}$$

Should the scale of the objective resolution be extended in detail (i.e., in smallness)--as made possible, for example, by a finer grid array--then the noise component reduces and the relevant information weight of the measure is increased.

2.1.4 Addition of Information

Let $p_1, p_2, \ldots p_n, \ldots p_N$ be N independent estimates of an object value p. Let the respective weights be $A_1, A_2, \ldots A_n, \ldots A_N$. The best estimate of p that this information affords is expressed by

$$p^* = \frac{A_1 p_1 + A_2 p_2 + \ldots A_n p_n + \ldots A_N p_N}{A_1 + A_2 + \ldots A_n + \ldots A_N} \quad . \tag{6}$$

And the weight is expressed by

$$A^* = A_1 + A_2 + \ldots A_n + \ldots A_N \quad . \tag{7}$$

This assimilation can be performed sequentially, saving only the accrued value and its growing weight. The result is independent of the order of assimilation.

2.1.5 Removal of Information

The contribution of a piece of information, p_n of weight A_n, which has already been assembled into a resultant, p^* of weight A^*, can be removed by subtraction:

$$p_B = \frac{A^* p^* - A_n p_n}{A^* - A_n} \tag{8}$$

$$A_B = A^* - A_n \quad . \tag{9}$$

The information, p_B of weight A_B, represents the independent background (i.e., verifying) information relative to the nth report which had been assembled into the resultant.

For a report which was <u>withheld</u> from the assembly the total resultant, p^* of weight A^*, represents the independent background information:

$$p_B = p^* \quad , \quad A_B = A^* \quad . \tag{10}$$

The background information may be used to gross-error check and reevaluate any piece of information.

2.2 Gross-Error Check and Reevaluation

A piece of information is invalid if it is not what it purports to be. If the reported value differs widely from that measured, or if the reported location of the measurement is considerably removed from the true one, then the report is termed a gross error. For an objective analysis technique to be acceptable it must have a well-developed capability for invalidating gross errors.

A piece of information which lies in the wings of the distribution implied by the associated variance is termed an outlier or maverick. It is a valid report but is not representative.

Gross errors may occur with any value. They are distinguishable only if the disparity with the background information is significant. Mavericks are always confused to some extent with gross errors.

Let p_n of weight A_n be a piece of information (e.g., the nth report) which is to be checked for validity and reevaluation (confirmation or weight reduction). Let p_B of weight A_B be the independent background information relative to the same object--but unknown--value p. The actual difference between the estimates is

$$p_n - p_B \quad . \tag{11}$$

The associated variance, here denoted by $\sigma^2_{n,B}$, of this difference is

$$\sigma^2_{n,B} = A_n^{-1} + A_B^{-1} = \frac{A_B + A_n}{A_n \; A_B} \quad . \tag{12}$$

The actual difference measured in units of the associated standard difference, $\sigma_{n,B}$, is denoted by λ_n. This normalized difference is given by

$$\lambda_n^2 = \left(p_n - p_B\right)^2 / \sigma_{n,B}^2 = \frac{A_B \; A_n}{A_n + A_B} \left(p_n - p_B\right)^2 \quad . \tag{13}$$

The purported weight, A_n, is reevaluated on the basis of λ_n. The reevaluated weight is expressed

$$A_{nR} = \begin{cases} A_n & \text{for } \lambda_n^2 \leqq 1 \\ \dfrac{2}{1+\lambda_n^2} A_n & \text{for } 1 < \lambda_n^2 \leqq \overline{\lambda^2} \\ 0 & \text{for } \lambda_n^2 > \overline{\lambda^2} \end{cases} \quad . \tag{14}$$

For $\lambda_n^2 \leqq 1$ the nth report is deemed to be acceptable as purported.

For $1 < \lambda_n^2 < \overline{\lambda^2}$ the nth report is deemed to be valid, but in order to reduce the influence of mavericks the weight is reduced.

For $\lambda_n^2 > \overline{\lambda^2}$ the nth report is deemed to be a gross error and is rejected as worthless.

The specification of $\overline{\lambda^2}$ is somewhat arbitrary. The specification is based, in application, on examinations of distributions of λ^2 for sample populations of reports. This procedure is referred to as tuning the parameter.

2.3 Propagation of Information

2.3.1 Explicit Blending

		Values	Weights
1	o	p_1	A_1
	–	$\mu_1 \Rightarrow p_2 - p_1$	B_1
2	o	p_2	A_2
	–	$\mu_2 \Rightarrow p_3 - p_2$	B_2
3	o	p_3	A_3
	–	$\mu_3 \Rightarrow p_4 - p_3$	B_3
4	o	p_4	A_4

Fig. 4 A one-dimensional array of independent information

Consider the implications of the distribution of independent information shown in Fig. 4. The pressure information at any one point generally has inferences at the other points via information in the form of estimates, μ, of the pressure difference between successive points. The estimates, μ, are finite-difference expressions of the first derivative of pressure along the axis of the array.

What is the resultant pressure, p^*, and the resultant weight, A^*, say at level 4, that is implied by the available information?

Four estimates are available for p_4:

Value	Weight
$p_1 + \mu_1 + \mu_2 + \mu_3$	$\left(A_1^{-1} + B_1^{-1} + B_2^{-1} + B_3^{-1}\right)^{-1}$
$p_2 + \mu_2 + \mu_3$	$\left(A_2^{-1} + B_2^{-1} + B_3^{-1}\right)^{-1}$
$p_3 + \mu_3$	$\left(A_3^{-1} + B_3^{-1}\right)^{-1}$
p_4	A_4

However it would be wrong to combine these by the rule for adding independent estimates--because they are not independent. The errors are correlated; there are common contributions to the variances of the first three estimates listed.

There is a stepwise procedure for combining the available information which satisfies the condition of independence. The information which propagates toward point 4 can be accrued step by step. At point 2 the values

$$p_1 + \mu_1 \quad \text{of weight} \quad \left(A_1^{-1} + B_1^{-1}\right)^{-1}$$

and

$$p_2 \quad \text{of weight} \quad A_2$$

can be combined by the rule for addition of independent estimates:

$$P_{2(1+2)} = \frac{\left(A_1^{-1} + B_1^{-1}\right)^{-1}\left(p_1 + \mu_1\right) + A_2\, p_2}{\left(A_1^{-1} + B_1^{-1}\right)^{-1} + A_2} \tag{15}$$

$$A_{2(1+2)} = \left(A_1^{-1} + B_1^{-1}\right)^{-1} + A_2 \quad . \tag{16}$$

The subscript parentheses have been added to enclose the sequence of combination.

Next, at point 3, the values

$$p_{2(1+2)} + \mu_2 \quad \text{of weight} \quad \left(A_{2(1+2)}^{-1} + B_2^{-1}\right)^{-1}$$

and

$$p_3 \quad \text{of weight} \quad A_3$$

can be combined to form

$$p_{3(1+2+3)} \quad \text{of weight} \quad A_{3(1+2+3)} \quad .$$

The procedure is repeated to form

$$p_{4(1+2+3+4)} \quad \text{of weight} \quad A_{4(1+2+3+4)} \quad ,$$

which are the desired resultants, p_4^* and A_4^* .

The information which arrives at point 4 from the one direction is the value

$$p_{3(1+2+3)} + \mu_3 \quad \text{of weight} \quad \left(A_{3(1+2+3)}^{-1} + B_3^{-1}\right)^{-1} \quad .$$

The weight is upper-bounded by $A_{3(1+2+3)}$ and, more significantly, by B_3 .

For an intermediate point, say point 2, the ambient information arrives from two directions. The available independent estimates are

$$p_1 + \mu_1 \quad \text{of weight} \quad \left(A_1^{-1} + B_1^{-1}\right)^{-1}$$

$$p_{3(4+3)} - \mu_2 \quad \text{of weight} \quad \left(A_{3(4+3)} + B_2^{-1}\right)^{-1}$$

$$p_2 \quad \text{of weight} \quad A_2 \quad .$$

This stepwise procedure can be applied to any arrays in which each point is connected to any other point by a <u>single path</u>. The resultant at point 6 of Fig. 5a, for example, can be calculated as follows: The information from points 1, 3+2, and 4 are combined with that at point 5 for an accrual value and weight at point 5. This accrual at point 5 is then combined, at point 6, with the information from points 7 and 8, and that at 6, giving the resultant at point 6.

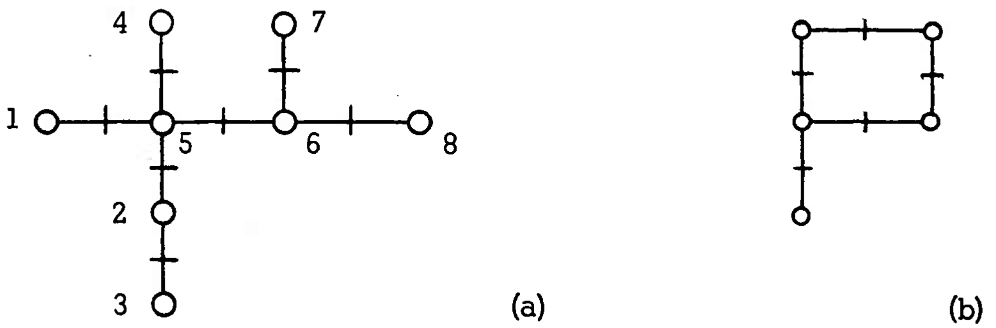

Fig. 5 Example of (a) a single-path array, and (b) a multi-path array.

This stepwise procedure is termed explicit blending.

The explicit blending procedure calculates resultant values and resultant weights.

Explicit blending cannot be applied to multi-path arrays.

2.3.2 Related Applications

2.3.2.1 Conceptual Basis

Explicit blending is useful for combining estimates of values and first differences along one dimension. It has found application in merging new data with climatology, at depths, for rendering resultant ocean thermal structure*. While not equal to the present primary task of objective analysis for two-dimensional arrays, including higher-order difference information, explicit blending forms part of the conceptual basis. It also arises in several related considerations in sea-level pressure analysis.

2.3.2.2 Collections of Pressure Reports with Reduction to Sea Level

A related consideration arises in the collection of pressure reports in the realm of a single collecting point (i.e., a grid reference location for pressure) in regions where terrain elevation is significant. The direct, but inconsistent, scheme which is generally followed is to reduce each station pressure, separately, into a sea-level pressure report, by some formula for the fictitious pressure addition associated with the drop in elevation to sea level. Each such reduced sea-level pressure report has a variance made up of the station-report variance and a variance addition for the reduction to sea level. The translation of each of these reduced sea-level pressure reports to the nearest grid reference location for pressure (See Fig. 2)--using available information as to the local shape of the

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*Mendenhall, Bruce R., 1970; "Design of a structure parameterization method for application to the three-dimensional analysis of ocean temperature", Final Report, Contract No. N00228-69-C-0833, (Fleet Numerical Weather Central), Meteorology International Incorporated, Monterey, California, 57 pp.

sea-level pressure field--adds another contribution of variance. The addition of the collection of such reduced reports by the rule for information addition is, generally, in violation of the condition of independence of estimates. As more and more reports are collected, the accrued weight can grow without bound even though the minimum terrain elevation may be thousands of feet in the region. This wrong method is schematically depicted by Fig. 6a.

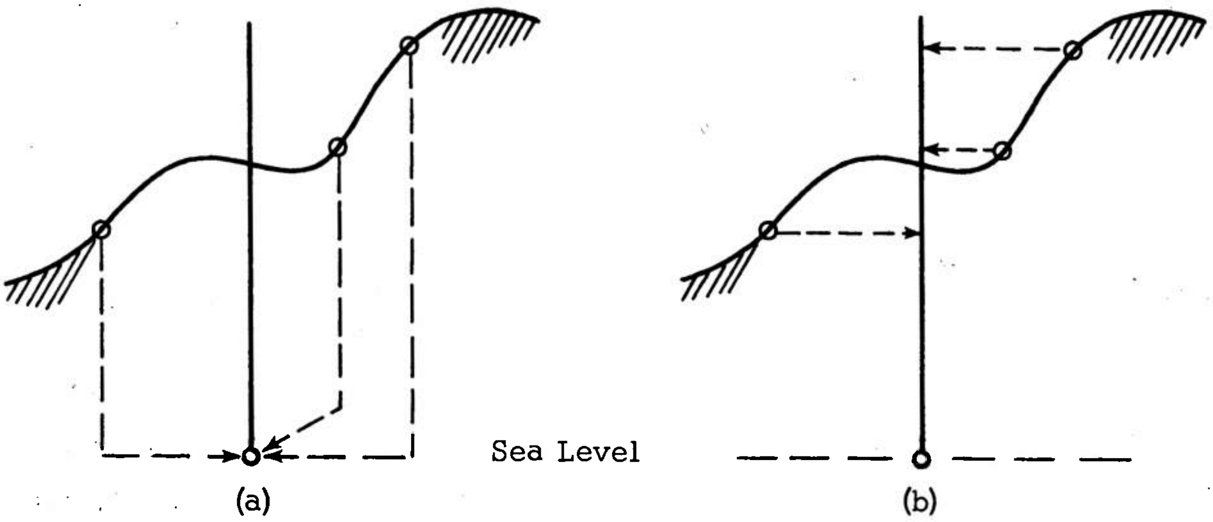

Fig. 6 Collection of Station Pressures at Sea Level

The correct method is schematically depicted by Fig. 6b. Each station pressure should first be projected, at station elevation, to the nearest grid reference location in the horizontal. This projection involves a variance contribution. All projected reports then pertain to elevations along the vertical at the grid reference location. The resultant at sea level can then be calculated by stepwise-downward explicit blending to sea level. The minimum variance in the resultant is that associated with the reduction from the lowest station elevation to sea level; no matter how many reports enter, the total weight is thereby bounded.

We are currently using the inconsistent scheme, depicted by Fig. 6a, because station pressures are not generally transmitted. They are reduced to sea level at the station for transmission. Formulas used for reduction to sea level differ from country to country and even within countries. Recovery of station pressure from the reduced sea-level pressure and station elevation is not generally feasible.

We are considering a refinement which will bound the total collected weight. According to this refinement each station report weight is reduced by adding variance associated with reduction down to the elevation of the lowest station in the collection; variance is also added in association with the horizontal projection to the grid reference location. The reports so weighted are combined. The total weight is then reduced by adding a variance associated with the remaining reduction to sea level. Adaptation of this refinement also involves some modification of the gross-error checking and reevaluation procedures.

2.3.2.3 Collections of Wind Reports

Similar considerations arise in the explicit use of wind observations. Each wind may be transformed into a pressure gradient estimate (two components) via a balance approximation. The associated variance is a combination of that equivalent to the variance in the wind estimate plus the contribution inherent in the balance approximation.

Consider a collection of wind reports in the realm of a single collecting point for a gradient component. There is a wrong way and a right way for combining the information into a gradient estimate.

The wrong way is to combine the transformed winds as estimates of the gradient component, with weights which have each been reduced for the imbalance contribution of variance. The more winds there are in the collection, the greater the apparent resolution of the gradient component, for the weight accrues without bound. This is unrealistic because there must remain the unknown imbalance between wind and pressure. The fault lies in the fact that the gradient estimates are not independent: they share the common variance contribution inherent in the balance approximation.

The consistent way to combine the gradient information in a collection of winds is as follows: Each wind is transformed by the balance approximation into an <u>equivalent</u> pressure gradient, but the variance is still regarded as only that contributed by the wind report variance now expressed in the equivalent pressure gradient units. The collection of winds, expressed as equivalent pressure gradients, are combined with the weights which reflect only equivalent wind variance. This gives the resolution of the wind expressed as an equivalent pressure gradient with weight in the same units. The weight is unbounded as an expression of wind resolution. The last step is inherent in assuming this equivalent pressure gradient to be an <u>estimate</u> of the pressure gradient. This assumption involves the addition of the variance inherent in the balance approximation. The resultant weight of the gradient estimate is thereby upper bounded.

Strictly considered, the wind information so collected at one grid reference location is not fully independent of that collected at a nearby reference location. The variance contributed by the imbalance is spatially correlated. This is one of the inconsistencies we have accepted, and for which partial allowances are made.

2.3.2.4 Collections of Off-Time Reports

The design of schemes for using off-time reports is similarly loaded with inconsistency traps.

Consider a collection of off-time pressure reports in the realm of a single grid reference location. Each report provides an estimate, at analysis time, with weight reduced by the addition of variance growth in the time disparity. The collection, however, cannot be combined with these reduced weights, because the off-time variance growth is correlated between reports. The consistent way is to explicitly blend the reports along the time axis at the grid reference location, as was proposed for the vertical axis in collecting pressure reports with reduction to sea level, in Section 2.3.2.2.

Such an explicitly blended collection of off-time reports at one reference location is not fully independent of such a collection at a nearby reference location. The variance growth factor is spatially correlated. For short time periods this correlation can be ignored. As the period approaches that of the analysis cycle the problem transforms into that of generating first-guess fields (i.e., information carried along the time axis) from prior analyses and predictions therefrom.

2.4 Implicit Blending

2.4.1 Formulations

The FIB technique includes an assembly operation, followed by a blending operation. In the assembly operation each new piece of independent information is combined with other independent information (including first-guess information) at the nearest reference location for the parameter. The basic parameters and reference locations are shown in Fig. 2 for the Sea-Level Pressure version. The array of assembly locations serves as a repository for the independent information in this assembly operation.

The analysis is effected by the blending operation. The object of the blending operation is to determine the best resultant field, p^*, with associated resultant weight field, A^*. The concept may be expressed by

$$A^*_{\ell,m}\, p^*_{\ell,m} = \underbrace{A_{\ell,m}\, p_{\ell,m}}_{\text{Independent information assembled at the reference location}} + \underbrace{A^a_{\ell,m}\, p^a_{\ell,m}}_{\text{Independent ambient information}}$$

$$A^*_{\ell,m} = A_{\ell,m} + A^a_{\ell,m} \quad . \tag{17}$$

These resultants are derived at every reference location for pressure. The independent ambient information stems from all reference locations for pressure other than ℓ,m and is propagated by gradient and higher-order difference information.

The pressure information at reference locations is connected by multiple paths and multiple finite-difference parameters. Explicit blending is not equal to the task of effecting the consistent blending. An implicit blending scheme is required.

Early efforts* to formulate a consistent implicit blending scheme proved inadequate. Later, we devised and tested a formulation which has stood up to all available tests for consistency. This made possible development of the first FIB version, that for sea-surface-temperature analysis**.

The formulation is based on minimizing the pertinent error functional--the total weighted disparity with all available elements of information.

The basic error functional for sea-level-pressure analysis is defined as

$$E \equiv \sum_{\ell,m} \Big\{ A_{\ell,m}\left(p^*_{\ell,m} - p_{\ell,m}\right)^2$$

$$+ B_{\ell,m}\left(p^*_{\ell,m+1} - p^*_{\ell,m} - \mu_{\ell,m}\right)^2$$

$$+ C_{\ell,m}\left(p^*_{\ell+1,m} - p^*_{\ell,m} - \nu_{\ell,m}\right)^2$$

$$+ D_{\ell,m}\left(p^*_{\ell,m+1} + p^*_{\ell+1,m} + p^*_{\ell,m-1} + p^*_{\ell-1,m} - 4p^*_{\ell,m} - L_{\ell,m}\right)^2 \Big\} \quad . \qquad (18)$$

The summation extends over all area modules. The elements of information are referred to the locations shown in Fig. 2. The boundary modules are not distinguished except that weights of all those information elements which couple with the exterior are identically zero. Each objective resultant element of p^* is that which minimizes its contribution to E.

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*Tilden, Charles E., James R. Clark and Manfred M. Holl, 1969; "Techniques for the utilization of satellite observations in operational mapping of sea surface temperature, Final Report, Contract No. N62306-C-0288 (Naval Air Systems Command), Meteorology International Incorporated, Monterey, California, 33 pp.

**Ibid, p. 4

2.4.2 The Blending System of Linear Equations

The arbitrary ℓ,m component of the blending system of linear equations is given by setting

$$\frac{\partial E}{\partial p^*_{\ell,m}} = 0 \quad . \tag{19}$$

This gives

$$\begin{aligned}
& S_{\ell,m}\, p^*_{\ell,m} \qquad - \qquad A_{\ell,m}\, p_{\ell,m} \\
& - B_{\ell,m}\left(p^*_{\ell,m+1} - \mu_{\ell,m}\right) - B_{\ell,m-1}\left(p^*_{\ell,m-1} + \mu_{\ell,m-1}\right) \\
& - C_{\ell,m}\left(p^*_{\ell+1,m} - \nu_{\ell,m}\right) - C_{\ell-1,m}\left(p^*_{\ell-1,m} + \nu_{\ell-1,m}\right) \\
& - 4\, D_{\ell,m}\left(p^*_{\ell,m+1} + p^*_{\ell+1,m} + p^*_{\ell,m-1} + p^*_{\ell-1,m} - L_{\ell,m}\right) \\
& - D_{\ell,m-1}\left(4\, p^*_{\ell,m-1} + L_{\ell,m-1} - p^*_{\ell+1,m-1} - p^*_{\ell,m-2} - p^*_{\ell-1,m-1}\right) \\
& - D_{\ell-1,m}\left(4\, p^*_{\ell-1,m} + L_{\ell-1,m} - p^*_{\ell-1,m+1} - p^*_{\ell-1,m-1} - p^*_{\ell-2,m}\right) \\
& - D_{\ell,m+1}\left(4\, p^*_{\ell,m+1} + L_{\ell,m+1} - p^*_{\ell,m+2} - p^*_{\ell+1,m+1} - p^*_{\ell-1,m+1}\right) \\
& - D_{\ell+1,m}\left(4\, p^*_{\ell+1,m} + L_{\ell+1,m} - p^*_{\ell+1,m+1} - p^*_{\ell+2,m} - p^*_{\ell+1,m-1}\right) \\
& = 0 \quad ,
\end{aligned} \tag{20}$$

where

$$S_{\ell,m} \equiv A_{\ell,m} + B_{\ell,m} + B_{\ell,m-1} + C_{\ell,m} + C_{\ell-1,m}$$

$$+ 16\,D_{\ell,m} + D_{\ell,m-1} + D_{\ell-1,m} + D_{\ell,m+1} + D_{\ell+1,m} \; . \tag{21}$$

The system of linear equations may be expressed in matrix notation:

$$\underset{\approx}{M}\;\underset{\sim}{p}^* = \underset{\sim}{G} \; . \tag{22}$$

The arbitrary row of the coefficient matrix $\underset{\approx}{M}$, that row corresponding to the ℓ,m element, may be exhibited in stencil form, acting on the two-dimensional grid array of reference locations for the pressure values:

		$+D_{\ell,m+1}$		
	$+D_{\ell,m+1}$ $+D_{\ell-1,m}$	$-4D_{\ell,m+1}$ $-B_{\ell,m}-4D_{\ell,m}$	$+D_{\ell,m+1}$ $+D_{\ell+1,m}$	
$+D_{\ell-1,m}$	$-4D_{\ell-1,m}$ $-C_{\ell-1,m}-4D_{\ell,m}$	$S_{\ell,m}$	$-4D_{\ell+1,m}$ $-C_{\ell,m}-4D_{\ell,m}$	$+D_{\ell+1,m}$
	$+D_{\ell-1,m}$ $+D_{\ell,m-1}$	$-B_{\ell,m-1}-4D_{\ell,m}$ $-4D_{\ell,m-1}$	$+D_{\ell+1,m}$ $+D_{\ell,m-1}$	
		$+D_{\ell,m-1}$		

The corresponding element of the forcing vector $\underset{\sim}{G}$ is

$$
\begin{aligned}
G_{\ell,m} \equiv {} & + A_{\ell,m} p_{\ell,m} - B_{\ell,m}\mu_{\ell,m} + B_{\ell,m-1}\mu_{\ell,m-1} \\
& - C_{\ell,m}\nu_{\ell,m} + C_{\ell-1,m}\nu_{\ell-1,m} \\
& - 4D_{\ell,m}L_{\ell,m} + D_{\ell,m-1}L_{\ell,m-1} + D_{\ell-1,m}L_{\ell-1,m} \\
& + D_{\ell,m+1}L_{\ell,m+1} + D_{\ell+1,m}L_{\ell+1,m} \quad . \qquad (23)
\end{aligned}
$$

The matrix $\underset{\approx}{M}$ is symmetric positive definite.

The formal solution for p^* is expressed by matrix inversion:

$$\underset{\sim}{p}^* = \underset{\approx}{M}^{-1} \underset{\sim}{G} \quad . \tag{24}$$

However since $\underset{\approx}{M}$, for a 125x125 grid array, has dimensions $125^2 x 125^2$, the inversion of $\underset{\approx}{M}$ is not a feasible calculation. Recourse must be made to iterative methods for arriving at approximate solutions for p^*. These methods are referred to as relaxation techniques.

The blending solution, as exhibited so far, includes no explicit reference for the determination of the resultant weight field, $\underset{\sim}{A}^*$. However, the fact that the resultant p^* elements are specified by the blending system implies that A^* is also specified, and apparently this specification is implicit in the blending system. This conclusion assumes that the blending system is conceptually consistent. These matters require some sleuthing, schematic analysis, and verifications.

2.4.3 Implied Specification of A^*

Equation (24) states that, for an arbitrary ℓ,m component,

$$p^*_{\ell,m} = M_{\ell,m;\ell,m} G_{\ell,m} + \sum_{i,j \neq \ell,m} M_{\ell,m;i,j} G_{i,j} \tag{25}$$

where $M_{\ell,m;\ell,m}$ is the diagonal element, and the $M_{\ell,m;i,j}$ $(i,j \neq \ell,m)$ are the off-diagonal elements, of the ℓ,m row of the inverse matrix, $\underset{\approx}{M}^{-1}$. By separating $G_{\ell,m}$ into two parts,

$$G_{\ell,m} \equiv A_{\ell,m} p_{\ell,m} + H_{\ell,m} \quad . \tag{26}$$

Eq. (25) can be expressed in the form

$$\begin{aligned} p^*_{\ell,m} &= M_{\ell,m;\ell,m} A_{\ell,m} p_{\ell,m} \\ &+ M_{\ell,m;\ell,m} H_{\ell,m} + \sum_{i,j \neq \ell,m} M_{\ell,m;i,j} G_{i,j} \quad . \end{aligned} \tag{27}$$

If we have obtained the desired resultant which is schematically expressed by Eq. (17), then the implications are that

$$M_{\ell,m;\ell,m} A_{\ell,m} p_{\ell,m} \equiv A_{\ell,m} p_{\ell,m} / A^*_{\ell,m} \tag{28}$$

and

$$\begin{aligned} M_{\ell,m;\ell,m} H_{\ell,m} &+ \sum_{i,j \neq \ell,m} M_{\ell,m;i,j} G_{i,j} \\ &\equiv A^a_{\ell,m} p^a_{\ell,m} / A^*_{\ell,m} \quad . \end{aligned} \tag{29}$$

According to Eq. (28) the elusive A^* elements are given by the diagonal elements of the inverse matrix:

$$A^*_{\ell,m} \equiv M^{-1}_{\ell,m;\ell,m} \quad . \tag{30}$$

Inspection of the form of Eqs. (22) and (24) and of the functional dependencies of the matrix elements reveals compatability with this conclusion. However, additional support is desirable.

2.4.4 The Calculation of A^*

For large grid arrays it is generally not feasible to attempt the inversion of the matrix $\underset{\approx}{M}$. And, it is just about as unrealistic to attempt a frontal attack on solving only for the diagonal elements of the inverse, $\underset{\approx}{M}^{-1}$.

The conceptual relationship expressed by Eq. (17) suggests a practical approach--optimizing between accuracy and computation costs. According to Eq. (17) a finite change in $p_{\ell,m}$, keeping all other forcing components and all weights fixed, results in a corresponding finite change in $p^*_{\ell,m}$ as expressed by

$$A^*_{\ell,m}\,\delta p^*_{\ell,m} = A_{\ell,m}\,\delta p_{\ell,m} \quad . \tag{31}$$

The scheme is applied after obtaining a complete resultant distribution of p^* , of adequate accuracy.

The scheme, implied by Eq. (31), is applied for the determination of A^* at one reference location at a time. Each such application involves re-solving for the field of p^* . At first glance this implies an inordinate amount of computing. However, the region of influence of $p_{\ell,m}$ on the p^*

field, diminishes with distance from the ℓ,m location. Hence p^* need be re-solved, subject to the specified change $\delta p_{\ell,m}$, only for a limited region about the ℓ,m location.

In areas abundant with A weight, the region may be quite limited. In other areas, where A weight is sparse, the region should be more extended to obtain comparable accuracy.

The re-solving of p^*, subject to $\delta p_{\ell,m}$, determines $\delta p^*_{\ell,m}$. And, according to Eq. (31),

$$A^*_{\ell,m} = A_{\ell,m}\ \delta p_{\ell,m} / \delta p^*_{\ell,m} \quad . \tag{32}$$

A similar scheme can be used to calculate the combined weight of the other parameters whose values are defined by p^*. For example,

$$\mu^*_{\ell,m} \equiv p^*_{\ell,m+1} - p^*_{\ell,m} \quad . \tag{33}$$

Consider $\mu^*_{\ell,m}$ to be the resultant according to

$$B^*_{\ell,m}\ \mu^*_{\ell,m} = B_{\ell,m}\ \mu_{\ell,m} + B^a_{\ell,m}\ \mu^a_{\ell,m} \quad . \tag{34}$$

Re-solving for p^*, subject to the single finite change $\delta\mu_{\ell,m}$, determines the corresponding finite change $\delta\mu^*_{\ell,m}$. Accordingly,

$$B^*_{\ell,m} = B_{\ell,m}\ \delta\mu_{\ell,m} / \delta\mu^*_{\ell,m} \quad . \tag{35}$$

2.4.5 Additional Support for the Blending Formulation

Additional evidence that the desired blending is effected by the developed formulation for implicit blending is as follows:

Numerical tests verify that this implicit blending produces the same parameter values, with the same weights, as are produced by explicit blending in single-path arrays.

Numerical tests verify that for any arrays the ambient contribution, $p^a_{\ell,m}$ of weight $A^a_{\ell,m}$, stemming from all reference locations other than ℓ,m, is independent of $p_{\ell,m}$ and of its weight $A_{\ell,m}$.

2.5 Additional Spreading Parameters

2.5.1 Expansion Terms

Additional higher-order difference parameters may be added to the blending scheme in order to enhance the spreading of independent information and in order to effect additional controls on the character of the resulting analysis.

Additional parameters are suggested by terms of the extrapolation expansion:

$$p = p_o + \delta \mathbb{r} \cdot \nabla p + \frac{1}{2} \delta \mathbb{r} \cdot \nabla \delta \mathbb{r} \cdot \nabla p + \ldots \tag{36}$$

where the displacement is

$$\delta \mathbb{r} \equiv \mathbb{i}\, \delta x + \mathbb{j}\, \delta y \quad .$$

The terms may be expressed in Cartesian coordinates:

$$\delta \mathbb{r} \cdot \nabla p \equiv \delta x \frac{\partial p}{\partial x} + \delta y \frac{\partial p}{\partial y} \tag{37}$$

$$\delta \mathbb{r} \cdot \nabla \delta \mathbb{r} \cdot \nabla p \equiv \delta x^2 \frac{\partial^2 p}{\partial x^2} + \delta y^2 \frac{\partial^2 p}{\partial y^2} + 2\, \delta x \delta y \frac{\partial^2 p}{\partial x \partial y} \quad . \tag{38}$$

This expansion suggests that the second derivatives and the cross derivative are significant in the spreading of information.

2.5.2 Second-Difference Parameters

The second differences are defined by the parameters

$$\hat{\mu}_{\ell,m} \Rightarrow p_{\ell,m+1} + p_{\ell,m-1} - 2\,p_{\ell,m} \tag{39}$$

$$\hat{\nu}_{\ell,m} \Rightarrow p_{\ell+1,m} + p_{\ell-1,m} - 2\,p_{\ell,m} \quad . \tag{40}$$

It is adequate for present purposes that the weights of relevant available $\hat{\mu}_{\ell,m}$ and $\hat{\nu}_{\ell,m}$ information be equal. Denote this weight by $F_{\ell,m}$. The contribution to the error functional is

$$\delta E = \sum_{\ell,m} \Big\{ F_{\ell,m} \left(p^*_{\ell,m+1} + p^*_{\ell,m-1} - 2\,p^*_{\ell,m} - \hat{\mu}_{\ell,m} \right)^2$$
$$+ F_{\ell,m} \left(p^*_{\ell+1,m} + p^*_{\ell-1,m} - 2\,p^*_{\ell,m} - \hat{\nu}_{\ell,m} \right)^2 \Big\} \quad . \tag{41}$$

The resulting additions to the system of blending equations may be expressed as contributions to Eq. (23):

The contribution to $S_{\ell,m}$, given by Eq. (21), is

$$\delta S_{\ell,m} \equiv 8\,F_{\ell,m} + F_{\ell,m-1} + F_{\ell,m+1} + F_{\ell-1,m} + F_{\ell+1,m} \quad . \tag{42}$$

		$+F_{\ell,m+1}$		
		$-2F_{\ell,m}-2F_{\ell,m+1}$		
$+F_{\ell-1,m}$	$-2F_{\ell,m}-2F_{\ell-1,m}$	$\delta S_{\ell,m}$	$-2F_{\ell,m}-2F_{\ell+1,m}$	$+F_{\ell+1,m}$
		$-2F_{\ell,m}-2F_{\ell,m-1}$		
		$+F_{\ell,m-1}$		

The contribution to the forcing element, $G_{\ell,m}$, is

$$\delta G_{\ell,m} \equiv -2F_{\ell,m}\hat{\mu}_{\ell,m} + F_{\ell,m-1}\hat{\mu}_{\ell,m-1} + F_{\ell,m+1}\hat{\mu}_{\ell,m+1}$$

$$-2F_{\ell,m}\hat{\nu}_{\ell,m} + F_{\ell-1,m}\hat{\nu}_{\ell-1,m} + F_{\ell+1,m}\hat{\nu}_{\ell+1,m} \quad . \tag{43}$$

2.5.3 Cross-Difference Parameter

The cross difference is defined by the parameter

$$\gamma_{\ell,m} \Rightarrow p_{\ell+1,m+1} + p_{\ell,m} - p_{\ell+1,m} - p_{\ell,m+1} \quad . \tag{44}$$

The weight is denoted by $K_{\ell,m}$. The contribution to the error functional is

$$\delta E = \sum_{\ell,m} \left\{ K_{\ell,m} \left(p^*_{\ell+1,m+1} + p^*_{\ell,m} - p^*_{\ell+1,m} - p^*_{\ell,m+1} - \gamma_{\ell,m} \right)^2 \right\} \quad . \tag{45}$$

The contribution to $S_{\ell,m}$ is

$$\delta S_{\ell,m} \equiv K_{\ell,m} + K_{\ell-1,m-1} + K_{\ell,m-1} + K_{\ell-1,m} \quad . \tag{46}$$

The contribution to the stencil on p^* is

$+ K_{\ell-1,m}$	$- K_{\ell-1,m}$ $- K_{\ell,m}$	$+ K_{\ell,m}$
$- K_{\ell-1,m}$ $- K_{\ell-1,m-1}$	$\delta S_{\ell,m}$	$- K_{\ell,m}$ $- K_{\ell,m-1}$
$+ K_{\ell-1,m-1}$	$- K_{\ell-1,m-1}$ $- K_{\ell,m-1}$	$+ K_{\ell,m-1}$

The contribution to the forcing element, $G_{\ell,m}$, is

$$\delta G_{\ell,m} \equiv K_{\ell,m}\gamma_{\ell,m} + K_{\ell-1,m-1}\gamma_{\ell-1,m-1} - K_{\ell,m-1}\gamma_{\ell,m-1} - K_{\ell-1,m}\gamma_{\ell-1,m} \quad . \tag{47}$$

2.5.4 The Spreading Parameters

We distinguish between the data parameters and the spreading parameters. The data parameters are the pressure, p , and the first differences, μ and ν . The spreading parameters are of higher order and include the Laplacian, L , the second differences, $\hat{\mu}$ and $\hat{\nu}$, and the cross difference, γ .

The information in the spreading parameters may be considered to include information derived from differences of these differences, that is, from terms of their analytic expansions--from higher-order terms to which there is no explicit reference because of truncation. Thus their weights do not represent independent information: the variances are spatially correlated. However this does not violate the blending conceptions. The information in the spreading parameters may be considered to be the resultant of a blending operation which assimilates all higher-order information, preparatory to the next step in downward blending which is actually performed. This conception defines blending to be an integrating procedure which may be performed stepwise in order.

The spreading values and weights have spatial continuity. In a sense they are analyzed fields. The local addition of information to a spreading parameter should be effected by a reanalysis, or restructuring, procedure based on the analytical continuity.

2.6 Relaxation Techniques

2.6.1 Solution of Blending System for p^*

The general method of Successive Over-Relaxation (SOR) is appropriate for the solution of the linear system expressed by Eq. (22). The matrix $\underset{\approx}{M}$ is normalized; this procedure makes each diagonal element unity. The matrix is factored:

$$\underset{\approx}{M} \equiv \underset{\approx}{I} + \underset{\approx}{U} + \underset{\approx}{V} \tag{48}$$

where $\underset{\approx}{I}$ is the identity matrix, $\underset{\approx}{U}$ has non-zero elements only above the main diagonal, and $\underset{\approx}{V}$ has non-zero elements only below the main diagonal. Equation (22) expands into

$$\underset{\sim}{p}^* = \underset{\sim}{G} - \underset{\approx}{U}\,\underset{\sim}{p}^* - \underset{\approx}{V}\,\underset{\sim}{p}^* \quad . \tag{49}$$

The direct SOR method is defined by

$$\underset{\sim}{p}^{*(r+1)} = \underset{\sim}{p}^{*(r)} + \omega\left(\underset{\sim}{G} - \underset{\approx}{U}\,\underset{\sim}{p}^{*(r)} - \underset{\approx}{V}\,\underset{\sim}{p}^{*(r+1)} - \underset{\sim}{p}^{*(r)}\right) \quad , \tag{50}$$

where the superscript in parentheses denotes the pass (i.e., iteration) number of the successive estimate. The method converges for $0 < \omega < 2$. Over-relaxation implies $\omega > 1$. We are still free to design the ordering of the elements of the vector $\underset{\sim}{p}^*$.

Of the orderings tested, the following ordering performed best--that is, it appeared to give the most rapid convergence.

The elements of $\underset{\sim}{p}^*$ are divided into five subsets. A subset includes every fifth element of each row of the grid-point array, with a displacement of two elements from one row to the next. The division is illustrated by Fig. 7.

1	4	2	5	3	1	4	2	5	3	1	4	.	.	.
5	3	1	4	2	5	3	1	4	2	5	3	.	.	.
4	2	5	3	1	4	2	5	3	1	4	2	.	.	.
3	1	4	2	5	3	1	4	2	5	3	1	.	.	.
2	5	3	1	4	2	5	3	1	4	2	5	.	.	.
1	4	2	5	3	1	4	2	5	3	1	4	.	.	.
5	3	1	4	2	5	3	1	4	2	5	3	.	.	.
4	2	5	3	1	4	2	5	3	1	4	2	.	.	.
.	.	.												

Fig. 7 Labelling, as to subset, of a section of the grid-point array.

The ordering of the vector $\underset{\sim}{p}^*$ is specified as a sequencing of the subsets:

$$\underset{\sim}{p}^* = \begin{pmatrix} \underset{\sim}{1} \\ \underset{\sim}{2} \\ \underset{\sim}{3} \\ \underset{\sim}{4} \\ \underset{\sim}{5} \end{pmatrix} . \tag{51}$$

The significance of this separation is that the ordering within each subset is immaterial. The calculation of an element is independent of the other elements of the same subset as that element. In effect, in computing each subset, calculated according to Eq. (50), the scheme is effectively both successive and simultaneous. However, in a full pass encompassing all five subsets, the scheme is definitely successive.

The first full pass is calculated with $\omega \equiv 1$. In the second pass ω is set high, $\approx$ 1.95, and is decreased with each successive pass. The last pass is made with ω set at 1.

2.6.2 Calculation of A^*

For each element, the calculation of $A^*_{\ell,m}$ involves a re-solution of p^* subject to an adjustment in the forcing element, $\delta p_{\ell,m}$. This re-solution is performed only for a limited region about the ℓ,m location. In areas abundant with A weight, the region may be quite limited. In other areas, where A weight is sparse, the region should be more extended to obtain comparable accuracy.

The method adopted for this re-solution of p^* is the SOR method but with $\omega \equiv 1$,--that is, we use Eq. (49):

$$\underset{\sim}{p}^{*(r+1)} = \underset{\sim}{G} - \underset{\approx}{U}\,\underset{\sim}{p}^{*(r)} - \underset{\approx}{V}\,\underset{\sim}{p}^{*(r+1)} \quad . \tag{52}$$

The ordering for the limited region is by sequencing as to subset, with subset identification according to Fig. 7.

Before beginning with the iterations, using Eq. (52), the first-guess for p^* is adjusted at all elements of the limited region, by the increment

$$\delta p^* = \frac{1}{A_B + A_{\ell,m}} \quad , \tag{53}$$

where A_B is a specified approximation of background weight.

The elements of p^* which surround the limited region are not changed in response to $\delta p_{\ell,m}$. This is a restraint on $\delta p^*_{\ell,m}$ and causes an overestimate of $A^*_{\ell,m}$. The first-guess adjustment, according to Eq. (53), introduces a compensating bias which is, however, dissipated if the limited region is taken very close to convergence.

2.7 The Time Axis

2.7.1 Explicit Use of First-Guess Fields

How should the value inherent in a first-guess field, predicted from the preceding analysis, be exploited? Wherein lies the information? In the gradients? In the Laplacian? For example, a 500-mb barotropic model may be quite skillful in predicting the Laplacian distribution, but may have considerably more variance in predicting the absolute height distribution. How should each of the parameter information fields be weighted?

It is easier to conceive of first-guess fields, for component parameters, to be weighted as to their resolution rather than as to independent worth. The resultant resolutions realized for the preceding analysis decrease with increasing variance growth due to error drift in prediction skills.

Resultant weights are appropriate for spreading parameters. However, first-guess fields for data parameters must be weighted according to independent worth for assimilation with the new observations.

If all parameters are derived from a single resultant prediction of the object parameter, then all the parameters, p, μ, ν, L, etc., are mutually consistent. However, such consistency is not essential.

The problem of determining the independent worth of first-guess fields for data parameters reduces to the following:

Given A_o^*, B_o^*, C_o^*, D_o^*, determine A_o, B_o, C_o.

This inversion conceivably gives rise to negative elements in the independent weights. This negative extension of information weight is formally tractable and presents no difficulties. We have not been able to solve this inversion problem except for trivial examples.

The procedure assumed for initial weighting of first-guess fields is assignment by specially designed formulas.

While expedient, this direct approach is a far cry from the comprehensive use which could be made of data along the time axis, if it were not for computer capacity limitations.

2.7.2 Comprehensive Approach

2.7.2.1 Explicit Use of Predicted Change

The comprehensive approach lies in the design of an error functional which couples several analysis times, e.g.,

$$t = \ldots (n-2)\tau , \quad (n-1)\tau , \quad n\tau \tag{54}$$

where τ is the period between analysis times. The blending is thereby extended to three dimensions.

At the new analysis time, $n\tau$, the error functional may couple the times $n\tau$, $(n-1)\tau$ and $(n-2)\tau$. The information which couples these analysis times is in the change fields, δp, $\delta\mu$, $\delta\nu$, etc., obtained from a prediction scheme calculating on earlier analyses from time $(n-2)\tau$ to $(n-1)\tau$ and from time $(n-1)\tau$ to $n\tau$. These predicted changes are appropriately weighted as data parameters and as spreading parameters.

The three-dimensional blending produces resultant analyses for times $(n-2)\tau$, $(n-1)\tau$ and $n\tau$. In an operational configuration the resultant for the latest time, $n\tau$, is the new current analysis. The resultant for the preceding time, $(n-1)\tau$, is an update analysis which includes off-time data both ahead and behind--achieving forward and backward continuity in time.

2.7.2.2 Implicit Dynamics

The comprehensiveness of the analysis system would reach ultimate levels by making the predicted changes, δp , $\delta\mu$, $\delta\nu$, etc., implicit functions of the analyses, running the governing dynamics both forward and backward in time. The resultant analyses would be free of the problem of update shock, having achieved optimum adjustments of continuity in time.

3. Additional Details of FIB/SLP

In this section further details of the FIB/SLP operations are discussed. A basic flow chart is shown in Fig. 8.

3.1 First-Guess Field Preparation

The first guess to the hemispheric sea-level pressure analysis is a combination of the applicable prognosis field and the previous hemispheric and tropical analyses extrapolated by a kinematical technique.

3.1.1 Combination of Previous Analyses

Grid point pressure values are first interpolated from the previous tropical analysis at each hemispheric grid point within the tropical grid (i.e., south of 60°N latitude). A weighted combination of the tropical interpolated and hemispheric values is made by the formula

$$p_o = wp_t + (1-w)\, p_h \tag{55}$$

where $w = 0.86 - \sin\varphi$, $(0 \leqq w \leqq 1)$,

φ is latitude,

p_h is the hemispheric grid point pressure value,

p_t is the interpolated tropical value, and

p_o is the resulting weighted value.

The formula gives a weight to the tropical value of zero at 60°N, 0.36 at 30°N, 0.86 at the equator and 1.0 from 8°S to the corners of the grid (19°S).

Use of the tropical analysis provides continuity with southern hemisphere data and, implicitly, with climatological information.

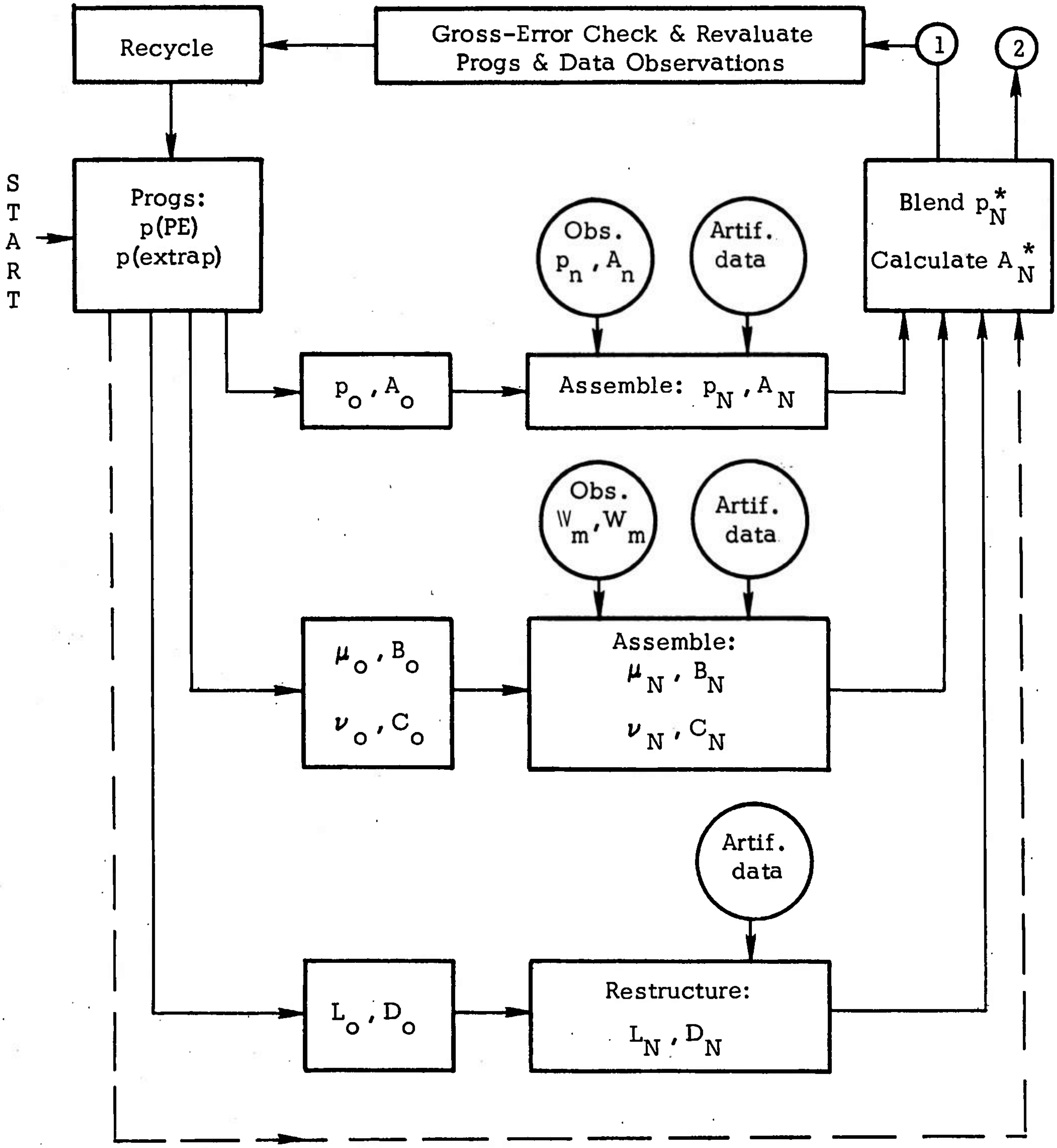

Fig. 8 Schematic Flow of Operations for FIB/SLP

3.1.2 Kinematical Extrapolation

Rather than utilize the six-hour-old combined analysis directly, an extrapolation is applied to minimize the effects of system movement. The residual 500-mb height analysis (SR500)--the actual 500-mb height analysis with the disturbance scale removed--is used as a steering field. From the SR500 height values geostrophic winds are used to derive six-hour movements. The program then looks upstream from each grid point in the six-hour-old combined surface analysis. The distance is specified by a percentage of the six-hour movement. The pressure interpolated there is brought back to the grid point. The optimum speed has been found to be about 58% of that specified by the SR500 geostrophic flow with a daily variation of about 10%. A continuing evaluation of the optimum percentage is made by the program.

3.1.3 Prognostic Field

The FNWC primitive-equation surface pressure forecast is used as the prognostic portion of the first guess. Depending on the analysis time, this is either a 6-hour or 12-hour forecast, while the kinematical extrapolation is always for a 6-hour period. These two fields are combined to produce the first guess (p_o) to FIB/SLP. The prognostic portion is weighted as a function of the sine of latitude so that in southern latitudes, the extrapolated field carries relatively more weight. The total weight of this field (A_o) increases from 0.001 at the north pole to 0.01 south of the equator. To account for the information in the tropical analysis outside the borders of the hemispheric grid, the weight values are increased on and one row inside the border.

3.1.4 Higher Derivative Fields

From the first-guess pressure field computed above, the first-guess finite difference (μ_o, ν_o), Laplacian (L_o), second-difference ($\hat{\mu}_o$, $\hat{\nu}_o$) and cross-difference (γ_o) fields are calculated from the formulas in Fig. 2 and Eqs. (39), (40) and (44). The Laplacian weight, D_o, is computed as

$$D_{o\ell,m} = \frac{1}{f_o + f_1 \overline{L_o^2}_{\ell,m}} \tag{56}$$

where $\overline{L_o^2}_{\ell,m}$ is the mean of L_o^2 at grid point (ℓ,m) and the four surrounding grid points and f_o and f_1 are adjustable constants (Appendix 1). The first-difference weights (B_o, C_o), second-difference weights (F_o) and cross-difference weights (K_o) are computed as constant factors of D_o. All weights are set to zero where their influence extends outside the grid, e.g., D_o at all boundary points, B_o along the top boundary, and C_o along the right side of the grid. ($B_o = C_o$ at all interior grid points.) The B_o and C_o values are increased where their respective μ and ν values refer to a boundary point; this extends the shape of the tropical field into the boundary regions of the hemispheric analysis.

3.2 Assembly of New Information

New pressure and wind data are assembled from ship (SH), land six-hourly synoptic (SM) and land hourly (SA) reports. The data lists are merged and sorted so that an hourly report will be used only if there is no synoptic report available at the same location. When the report at analysis time is missing, the one closest to the analysis time (up to $\pm$ 3 hours) is admitted to the data list along with an indication of report age. This may bring in three-hourly (SI) reports, but at considerably reduced weight, as discussed later.

3.2.1 Pressure Reports

The pressure data are reported in shortened form with the 100's of millibars omitted. The first step is to determine the multiple of 100mb that, when added to the report, will minimize the difference between the first guess and the modified report. As a result of this operation, no report will differ from the first guess by more than 50mb.

The pressure report can now be extrapolated to the nearest grid point, determined by rounding the exact i,j coordinates to the nearest integers. Extrapolation is made along the gradients of the best available field, i.e., the p_o field if the analysis is the first one being performed in the synoptic period, the result of first pass blending if in the second pass, or the final analysis for the same time period if performing an update analysis. The formula for extrapolating pressure reports assumes that the difference between the first guess and the report remains constant over the distance from the report to the grid point:

$$p_{\ell,m} = \left(p_o\right)_{\ell,m} + \left(p_n - p_o\right)_{i,j} \tag{57}$$

where $p_{\ell,m}$ is the extrapolated pressure at grid point (ℓ,m), $(p_o)_{\ell,m}$ is the guess value at the grid point, $(p_n)_{i,j}$ is the reported value at station location (i,j), and $(p_o)_{i,j}$ is the guess value interpolated at the station location using a Bessel 16-point operator.

The maximum allowable difference between the report and best available guess increases with the magnitude of the gradients in the vicinity of the report and with latitude. The report is considered a gross error and rejected if

$$\left| p_n - p_o \right|_{\ell,m} > f_2 + f_3 \left| \overline{g} \right|_{\ell,m} + f_4 \left(1 - \cos \varphi\right) \quad , \tag{58}$$

where $\overline{|g|}_{\ell,m}$ is the mean absolute gradient in the surrounding grid points, φ is latitude ($\leq 60^\circ$) and f_2, f_3 and f_4 are adjustable constants. In an area of strong gradients, north of 60°N latitude, the allowable tolerance may reach 30mb.

The reliability of a pressure report decreases with the magnitude of gradients in the vicinity of the report, with elevation, and with age. In terms of variance

$$\sigma_n^2 = f_5 + f_6\, \overline{g^2_{\ell,m}} + f_7 E^2 + \left(f_8 + f_9\, \overline{g^2_{\ell,m}}\right) H^2 \quad , \tag{59}$$

where σ_n^2 is the total variance of the pressure report, $\overline{g^2_{\ell,m}}$ is the mean square gradient surrounding the report, E is elevation, H is age and f_5 through f_9 are adjustable constants. The class of report is represented in f_5. The age term does not affect the intrinsic variance of a report, but does reduce its weight for assembly purposes. The weight, or reliability, of the report is $A_n = \sigma_n^{-2}$ where n refers to a single (unassembled) reliability. If several conflicting reports are received from the same station, their individual weights must be reduced since they are not independent information--their total weight should be A_n.

The assembled value of pressure at a grid point is a weighted mean of all data referred to the grid point, with each value contributing to the mean in accordance with its reliability. Such a weighted mean can be derived one report at a time, independent of assembly order, by

$$P_N = \left(P_{N-1} A_{N-1} + P_n A_n\right) / \left(A_{N-1} + A_n\right) \tag{60}$$

$$A_N = A_{N-1} + A_n \tag{61}$$

where p_n, A_n refer to a single report and P_N, A_N refer to the assembly of N reports.

The assembly is initialized with the first-guess pressure field (p_o, A_o). This must be distinguished from the best available field--which, at least partly, represents current information and cannot be considered independent. An example, considered later, is an analysis made earlier for the same time period. The first-guess pressure field is derived totally from past information, extrapolated forward in time, and is therefore independent information, validating the addition of variance rule (Section 2.1.2).

3.2.2 Wind Reports

In order to use a report of wind direction and speed, conversion must first be made to components of pressure finite differences along the grid coordinates by an appropriate balance equation. The balance equation is written in vector form as

$$\mathbb{V} \cdot \nabla \mathbb{V} + f\,\mathbb{k} \times \mathbb{V} + \varkappa \mathbb{V} = f\,\mathbb{k} \times \mathbb{V}_g \quad , \tag{62}$$

where $\mathbb{V}_g$ is the geostrophic wind, $\mathbb{V}$ is the actual wind, $\mathbb{k}$ is the unit vertical vector, $\varkappa$ is a frictional parameter, and f is the coriolis parameter. Local accelerations of the wind are omitted. The frictional effect on the surface wind is contained in the $\varkappa$ term. In component form the geostrophic wind may be written as

$$\begin{aligned} u_g &= R\left(u + \frac{\varkappa}{f} v\right) + \frac{1}{f}\left[u\left(\frac{\partial v}{\partial x}\right)_{g,o} + v\left(\frac{\partial v}{\partial y}\right)_{g,o}\right] \quad , \\ v_g &= R\left(v - \frac{\varkappa}{f} u\right) - \frac{1}{f}\left[u\left(\frac{\partial u}{\partial x}\right)_{g,o} + v\left(\frac{\partial u}{\partial y}\right)_{g,o}\right] \quad , \end{aligned} \tag{63}$$

where the derivative subscripts refer to geostrophic wind components in the best available field (e.g., an analysis for the same time) and u,v are the reported wind in component form. The value of R, slightly greater than 1.0, has been added to increase the difference between the geostrophic and actual velocities without increasing the inflow angle. This empirically accounts for the fact that, due to vertical momentum transfer, the frictional force is not directed exactly opposite the surface wind but rather opposite a direction somewhere between the surface and free flow.

The $\varkappa$ term allows for variations in roughness. It can be shown from Eq. (63), assuming negligible curvature, that $\varkappa = f \cdot \tan\alpha$ where α is the inflow angle of the surface wind. The value of $\varkappa$ is set for an approximate inflow angle of 12° over the sea and 22° over land.

The derivative terms are computed as four separate fields using a 16-point operator.* The values of the derivatives are then determined for each report by interpolation to exact station location.

The final pressure finite difference values, μ_n and ν_n, are computed by applying the geostrophic wind equation to the u_g and v_g components found above, and solving for pressure differences over 1 grid length in the m and ℓ grid directions, respectively.

The variance of the wind report is

$$\sigma_n^2 = f_{10} \sin^2\varphi \left(\frac{1 + \sin\varphi}{1.866}\right)^2 + f_6 \overline{g^2_{\ell,m}} + f_7 E^2 + \left(f_8 + f_9 \overline{g^2_{\ell,m}}\right) H^2 \qquad (64)$$

*Holl, M. M., 1966; "The assimilation of wind observations in the analysis of the atmospheric mass structure", Quarterly Report 1, Contract No. N00228-66-C-1923, (Fleet Numerical Weather Central), Meteorology International Incorporated, Monterey, California, 16 pp.

where f_{10} is a class variance. The first term on the right is the "calm wind" variance. The $\sin^2\varphi$ portion is needed to convert from a wind variance to a variance in terms of pressure difference and the remainder of the term accounts for variation in grid spacing with latitude. Note that a light wind in the tropics can have a very small variance when expressed in terms of an equivalent pressure difference. Since μ_n and ν_n are referred to the same grid point, the same weight can be assigned to each, i.e., $B_n = C_n = \sigma_n^{-2}$. Separate fields of B and C are maintained because of different boundary values on the sides where μ or ν would extend outside the grid.

The μ_n and ν_n pressure differences and their weight values ($B_n = C_n$) are assembled in the same manner as the pressure reports, beginning with first-guess fields μ_o, ν_o, B_o and C_o. No extrapolation is performed. The reference grid point which minimizes the μ and ν position error is found by subtracting 0.25 from the exact i and j coordinate and rounding the result. This offset is necessary when referring both differences to the same grid point considering the different locations of the μ and ν reference points (See Fig. 2).

No initial gross-error rejection is performed other than a limit on reported wind speed of 60m/sec.

Upon completion of the μ and ν assembly, the assembled weights are reduced at grid points containing data to account for the uncertainty in the balance equation conversion. (See Section 2.3.2.3.) Expressed as a variance:

$$\sigma_{BC}^2 = \sigma_a^2 + \sigma_b^2 \tag{65}$$

where σ_a^2 is the assembled variance at the grid point (i.e., the inverse of B or C), σ_b^2 is the variance of the balance approximation, and σ_{BC}^2

is the final variance at the grid point from which B and C are derived. Thus, the total weight at a grid point is bounded by the balance approximation variance, no matter how many wind reports are assembled at the grid point. This also reduces weights at grid points with single wind reports. The assembled reliabilities included variance in the wind report only. In terms of pressure difference, this variance may have been quite small, resulting in large assembled weight before reduction for the balance approximation.

3.3 Blending for p^*

When all data have been assembled into their respective fields, the program can blend the information into a best-fit p^* field which minimizes the error functional (Eqs. 18, 41, and 45). The requirement of minimizing the errors between the pressure field and its derivative fields at all grid points leads to the basic implicit blending equation (Eq. 20). The implicit nature of the problem requires an iterative solution. Considerable computational simplification is possible by combining terms which are constant throughout the iteration or which are common multiples of the same p^*. This leads to the forcing term (Eq. 23) which involves the starting fields only, and a sum term (Eq. 21) which concerns only p^* at the grid point being solved. Other terms involve p^* at surrounding grid points and can be grouped into a stencil operating on the p^* field. The inclusion of second-difference and cross-difference fields requires additional terms, as shown in Section 2.5.2 and 2.5.3. The total stencil is shown below:

<table>
<tr><td></td><td></td><td>$+D_{\ell,m+1}$
$+F_{\ell,m+1}$</td><td></td><td></td></tr>
<tr><td></td><td>$+D_{\ell-1,m}$
$+D_{\ell,m+1}$
$+K_{\ell-1,m}$</td><td>$-B_{\ell,m}$
$-4D_{\ell,m}-4D_{\ell,m+1}$
$-2F_{\ell,m}-2F_{\ell,m+1}$
$-K_{\ell-1,m}-K_{\ell,m}$</td><td>$+D_{\ell,m+1}$
$+D_{\ell+1,m}$
$+K_{\ell,m}$</td><td></td></tr>
<tr><td>$+D_{\ell-1,m}$
$+F_{\ell-1,m}$</td><td>$-C_{\ell-1,m}$
$-4D_{\ell-1,m}-4D_{\ell,m}$
$-2F_{\ell-1,m}-2F_{\ell,m}$
$-K_{\ell-1,m-1}-K_{\ell-1,m}$</td><td>$S_{\ell,m}$</td><td>$-C_{\ell,m}$
$-4D_{\ell,m}-4D_{\ell+1,m}$
$-2F_{\ell,m}-2F_{\ell+1,m}$
$-K_{\ell,m-1}-K_{\ell,m}$</td><td>$+D_{\ell+1,m}$
$+F_{\ell+1,m}$</td></tr>
<tr><td></td><td>$+D_{\ell-1,m}$
$+D_{\ell,m-1}$
$+K_{\ell-1,m-1}$</td><td>$-B_{\ell,m-1}$
$-4D_{\ell,m-1}-4D_{\ell,m}$
$-2F_{\ell,m-1}-2F_{\ell,m}$
$-K_{\ell-1,m-1}-K_{\ell,m-1}$</td><td>$+D_{\ell,m-1}$
$+D_{\ell+1,m}$
$+K_{\ell,m-1}$</td><td></td></tr>
<tr><td></td><td></td><td>$+D_{\ell,m-1}$
$+F_{\ell,m-1}$</td><td></td><td></td></tr>
</table>

By transforming into the following fields:

$$X_{\ell,m} \equiv C_{\ell,m} + 4D_{\ell,m} + 4D_{\ell+1,m} + 2F_{\ell,m} + 2F_{\ell+1,m} + K_{\ell,m-1} + K_{\ell,m} \quad ,$$

$$Y_{\ell,m} \equiv B_{\ell,m} + 4D_{\ell,m} + 4D_{\ell,m-1} + 2F_{\ell,m} + 2F_{\ell,m+1} + K_{\ell-1,m} + K_{\ell,m} \quad ,$$

$$Z_{\ell,m} \equiv D_{\ell,m+1} + D_{\ell+1,m} + K_{\ell,m} \quad ,$$

$$Q_{\ell,m} \equiv D_{\ell,m} + F_{\ell,m} \quad ,$$

$$R_{\ell,m} \equiv D_{\ell,m} + D_{\ell+1,m+1} + K_{\ell,m} \quad ,$$

the stencil reduces to

		$Q_{\ell,m+1}$		
	$R_{\ell-1,m}$	$-Y_{\ell,m}$	$Z_{\ell,m}$	
$Q_{\ell-1,m}$	$-X_{\ell-1,m}$	$S_{\ell,m}$	$-X_{\ell,m}$	$Q_{\ell+1,m}$
	$Z_{\ell-1,m-1}$	$-Y_{\ell,m-1}$	$R_{\ell,m-1}$	
		$Q_{\ell,m-1}$		

The sum term is:

$$S_{\ell,m} = A_{\ell,m} + B_{\ell,m} + B_{\ell,m-1} + C_{\ell,m} + C_{\ell-1,m}$$

$$+ 16\, D_{\ell,m} + D_{\ell,m-1} + D_{\ell-1,m} + D_{\ell,m+1} + D_{\ell+1,m}$$

$$+ 8\, F_{\ell,m} + F_{\ell,m-1} + F_{\ell-1,m} + F_{\ell,m+1} + F_{\ell+1,m}$$

$$+ K_{\ell,m} + K_{\ell-1,m-1} + K_{\ell,m-1} + K_{\ell-1,m} \quad , \tag{66}$$

and the forcing term is:

$$G_{\ell,m} = A_{\ell,m} p_{\ell,m} - B_{\ell,m} \mu_{\ell,m} + B_{\ell,m-1} \mu_{\ell,m-1} - C_{\ell,m} \nu_{\ell,m} + C_{\ell-1,m} \nu_{\ell-1,m}$$

$$-4D_{\ell,m} L_{\ell,m} + D_{\ell,m-1} L_{\ell,m-1} + D_{\ell-1,m} L_{\ell-1,m} + D_{\ell,m+1} L_{\ell,m+1} + D_{\ell+1,m} L_{\ell+1,m}$$

$$- 2F_{\ell,m} \hat{\mu}_{\ell,m} + F_{\ell,m-1} \hat{\mu}_{\ell,m-1} + F_{\ell,m+1} \hat{\mu}_{\ell,m+1}$$

$$- 2F_{\ell,m} \hat{\nu}_{\ell,m} + F_{\ell-1,m} \hat{\nu}_{\ell-1,m} + F_{\ell+1,m} \hat{\nu}_{\ell+1,m}$$

$$+ K_{\ell,m} \gamma_{\ell,m} + K_{\ell-1,m-1} \gamma_{\ell-1,m-1} - K_{\ell,m-1} \gamma_{\ell,m-1} - K_{\ell-1,m} \gamma_{\ell-1,m} \quad , \tag{67}$$

where $\hat{\mu}$ and $\hat{\nu}$ are the second differences in the m and ℓ directions, respectively (Eqs. (39) and (40)) and γ is the cross-difference (Eq. (44)). Thus, the blending operation requires eight 125x125 fields: X , Y , Z , Q , R , S , G and p^* .

Because of central memory storage limitations, these fields are maintained in extended core storage (ECS) and brought into central memory nine rows (125 points/row) at a time, throughout the iteration. (Only p^* need be stored in ECS after completing a set of rows.) Since grid-point values two rows away from the computation row are required, the nine rows include four rows of overlap. This yields five rows for actual computation, or 25 sets to complete the grid array. The p^* computation is made only every fifth point across a row in order to realize the maximum convergence rate (See Section 2.6.1). The starting point in each row follows the cycle (1, 3, 5, 2, 4). Five passes through the field are needed before p^* at every point has been modified once.

After the first five passes (representing one full pass), overrelaxation is applied to further improve the convergance rate. This is done by over-correcting p^*, that is, by changing p^* at a grid point by the computed change times an overrelaxation factor. This factor must be less than 2.0 to maintain computational stability. The factor is decreased from 1.95 at pass 6 to 1.0 in the final pass. This variation gives an even convergence rate over the spectrum of wavelengths encountered in the pressure field.

The first guess to the p^* iteration may be the assembled p_N field or another better field, if available. For example, when performing an update analysis, the previously blended analysis for the same time period may be used, which further enhances convergence. Note, however, that to avoid a compounding of information, such a field may be used only as a first guess to the iteration and may not be used in the forcing term.

Utilizing the methods discussed above to enhance the convergence rate, a total of 80 passes, operating on every fifth point, is found to be adequate. This is approximately equivalent, in computing time, to 16 passes when operating on every point. A well-converged p^* field is required for the next step--computation of the grid-point resultant reliability field, A^*.

3.4 Computation of A^*

The pressure reliabilities (A) assembled at each grid point represent only the reliability of the reports if they had no effect on each other, that is, if there were no blending or interaction between the grid points. The actual reliability of blended grid-point pressure values should always be greater than the assembled reliability to account for information propagated from surrounding grid points through gradient and higher-order information. For example, the reliability of a blended pressure value at a grid point with no assembled reports should be greater than the first-guess reliability if reports surround the grid point and if there is gradient knowledge.

A perturbation technique is used to compute A^* by measuring the leverage of a pressure change on the information at a grid point and its surrounding area. To obtain an exact solution of A^* it would be necessary at each grid point to re-solve for p^* over the entire field, given a fixed change of p in the forcing term. The value of A^* would then be

$$A^*_{\ell,m} = \frac{A_{\ell,m}\,\delta p_{\ell,m}}{\delta p^*_{\ell,m}} \tag{68}$$

as shown in Eq. (31). Stated in words, if the assembled pressure change (δp) has small leverage and changes p^* only slightly then δp^* will be small and A^* will be large.

To keep the program computation time within reasonable limits, the re-solution is carried out only in the vicinity of the grid point (ℓ,m) as shown in Fig. 9.

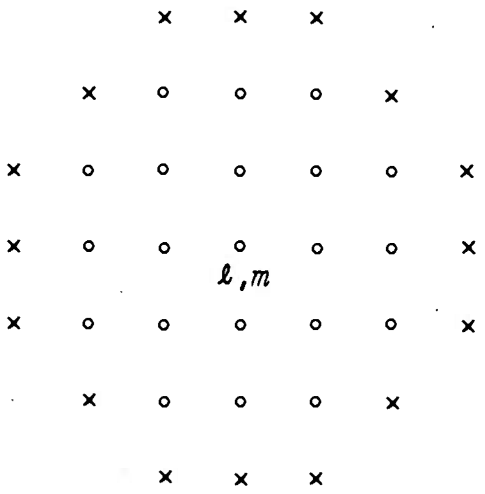

Fig. 9 Computational sub-area for $A^*_{\ell,m}$ indicated by (∘) grid points

Changes are made to p^* only at the (∘) points. At, and outside, the (x) points, no computation is performed when solving for A^* at grid point (ℓ,m). Holding p^* fixed at the (x) points is the same as assuming infinite reliability at these points. The effect is to limit, somewhat, the change in $p^*_{\ell,m}$ and therefore overestimate A^*. This overestimation is noticeable only in data-sparse areas or when no information other than first-guess is found within the (x) points. In these situations, the computational area could be enlarged to include at least some minimum amount of assembled information. Computation of A^* may extend to the grid boundaries. Points in the sub-area which are outside the grid do not affect the result since all weights extending across the boundaries have a value of zero.

The solution for A^* is found by applying the p^* formula to the fields used in the p^* computation except that p^* is now a blended field. The importance of a well-converged p^* field is seen by considering the A^* formula--the change in p^* should result only from the perturbation in p and not reflect further convergence in the original p^*.

The nine-row portions of the fields are sufficient for computing A^* along only one row, requiring 125 field exchanges. At each grid point (ℓ, m), p is changed in the forcing term only at (ℓ, m). Then p^* at all computation points is increased toward the expected result to speed convergence. The iteration for the modified p^* value is made through the area in a manner analagous to that used in the p^* blending.

2	5	3	1	4
1	4	2	5	3
5	3	$\underline{1}$	4	2
4	2	5	3	1
3	1	4	2	5

Fig. 10 Subset labelling in the $A^*_{\ell,m}$ computational sub-area. Grid point (ℓ, m) is underlined.

In Fig. 10, the #1 subset points are operated on first, then the #2 points, etc., through the #5 points. (The circled points are shown to complete the pattern only; no computations are made there.) This same pattern is repeated 4 times. As shown in Section 2.6.1, ordering within a subset is immaterial. In the last pass, the #5 point in the top row is skipped and a final computation is made at the center grid point (ℓ, m), underlined. (Note that the point skipped is not contained in the stencil for the center grid point.) $A^*_{\ell,m}$ may now be computed, knowing the $\delta p_{\ell,m}$ applied to the forcing term, the $A_{\ell,m}$ value and $\delta p^*_{\ell,m}$ computed as $(p^*_{\text{modified}} - p^*)_{\ell,m}$.

Before proceeding to the next grid point, the forcing term at (ℓ, m) is restored to its original value and A^* is packed with another field. The p^* values do not have to be restored since they are extracted for each point from the full p^* field which is never modified. Upon completion of a row, the packed A^* values are written in ECS.

No increase in the A^* computation area is attempted in data sparse regions. It has been found that far-removed data have only a small effect on A^* which is nearly masked by the background reliability. This background reliability results from the higher-order derivatives which have small weight at individual grid points but which propagate over long distances.

The grid point A^* values are needed in reevaluation which follows. In the first cycle of the program, A^* need be calculated only at data points, since the purpose of the first cycle is to evaluate the reports.

3.5 Reevaluation and Lateral Rejection

Knowledge of the blended pressure and resultant grid-point reliability enables the program to reevaluate, individually, any information that entered the analysis. By removing an individual datum value and its reliability from a grid point and comparing it with what remains, or the "background", the true reliability of the data may be quantitatively assessed (Section 2.2).

Removal of a report from the blended fields is the exact reversal of assembly:

$$A_B = A^* - A_a \quad , \tag{69}$$

$$p_B = \left(A^* p^* - A_a p_n\right) / A_B \quad , \tag{70}$$

where p_n is the report that was assembled with weight A_a and p_B is the background pressure, with weight A_B, that remains after removal of the report. The actual difference of these estimates is $p_n - p_B$ while the expected or standard difference is $\sigma_{n,B} = (A_n^{-1} + A_B^{-1})^{1/2}$, where A_n is the true weight of the total report. (Expected difference must be based on true report weight; the assembled weight, A_a, is reduced from A_n when different values are reported from the same station, and when a report is off-time.) By squaring the actual and expected differences, the ratio may be expressed in terms of variance:

$$\lambda_{np}^{\ 2} = \frac{A_B \ A_n}{A_B + A_n} \left(p_n - p_B\right)^2 \quad . \tag{71}$$

Some allowance may be made for disparity in position between p_n and p_B, accounting for the fact that reports occurring over a finite area must be expressed in a single grid-point value. This allowable disparity should be a function only of the gradient in the area of a grid point. If the λ^2 equation is written in terms of blended values only, this allowance may be made by reducing the difference $(p_n - p^*)^2$ for a portion of the p^* gradient. Substituting p^* and A^* in the $\lambda_{np}^{\ 2}$ equation:

$$\lambda_{np}^{\ 2} = \frac{\left(A^* - A_a\right) A_n}{A^* - A_a + A_n} \left[p_n - \frac{A^* p^* - A_a p_n}{A^* - A_a}\right]^2 \quad , \text{ or}$$

$$\lambda_{np}^{\ 2} = \frac{A_n \ A^{*2} \left(p_n - p^*\right)^2}{\left(A^* - A_a + A_n\right)\left(A^* - A_a\right)} \quad . \tag{72}$$

The mean square gradient per unit grid length around grid point (ℓ, m) is

$$\overline{g^2_{\ell,m}} = \left(\mu^{*\ 2}_{\ell,m} + \mu^{*\ 2}_{\ell,m-1} + \nu^{*\ 2}_{\ell,m} + \nu^{*\ 2}_{\ell-1,m}\right)/2 \quad . \tag{73}$$

A reasonable position disparity to allow is the r.m.s. distance of a large population of reports from their respective grid points, or $(d^2/6)^{1/2}$, where d is the grid spacing. Then $\overline{g^2_{\ell,m}}$ should be weighted by the mean square distance in unit grid spacing, or 1/6.

The final equation for λ_{np}^2 becomes

$$\lambda_{np}^2 = \frac{A_n A^{*2}\left[\left(p_n - p^*\right)^2 - \left(\mu^2_{\ell,m} + \mu^2_{\ell,m-1} + \nu^2_{\ell,m} + \nu^2_{\ell-1,m}\right)/12\right]}{\left(A^* - A_a + A_n\right)\left(A^* - A_a\right)} \geqq 0. \quad (74)$$

The value of λ_{np}^2 measures how accurate a pressure report actually is, as compared to its expected accuracy given by the reliability assigned to it. The median λ_{np}^2 for all reports in the analysis should therefore be approximately 1.0, indicating a balance between reports with more error and reports with less error than expected. This offers a method for checking the assignment of class reliabilities to reports, which otherwise would be arbitrary.

The value of λ_{np}^2 for a report can now be used for reevaluation. If $\lambda_{np}^2 \leqq 1$ the report is within its expected error and no change is made in its reliability. If $1 < \lambda_{np}^2 < \lambda^2_{maxp}$, where λ^2_{maxp} is the maximum allowed λ^2 for a pressure report, the report is reevaluated by the formula

$$A_{nR} = \frac{2\,A_n}{1 + \lambda_{np}^2} \quad (75)$$

where R indicates reevaluation. Any report of $\lambda_{np}^2 > \lambda^2_{maxp}$ is withheld from the final analysis cycle. Since this rejection is based on current information in the vicinity of the report, it may be termed a "lateral" reject as distinguished from the initial "gross error" reject.

Since blended difference reliabilities (B^*, C^*) are not produced, the reevaluation of wind reports must be done in a different manner. The expected variance of a wind report is approximately $1/B_n + \sigma_b^2$ where σ_b^2 is the variance added at the grid point for the balance approximation. (This is true only for a single wind report at a grid point since the balance variance is added only once, independent of the number of reports.) The variance ratio for a wind report is

$$\lambda_{nw}^2 = \frac{(\mu_n - \mu^*)^2 + (\nu_n - \nu^*)^2}{1/B_n + \sigma_b^2} \quad . \tag{76}$$

If $1 < \lambda_{nw}^2 < \lambda_{maxw}^2$ the report is reevaluated:

$$B_{nR} = C_{nR} = \frac{1}{\left(1/B_n + \sigma_b^2\right)\left(1 + \lambda_{nw}^2\right)/2 - \sigma_b^2} \quad . \tag{77}$$

The subtraction of σ_b^2 is necessary because it will be added back in the final assembly. As in the case of a pressure report, no change in reliability is made if $\lambda_{nw}^2 \leqq 1$ and the report is rejected if $\lambda_{nw}^2 > \lambda_{maxw}^2$. A vector sum and difference ratio is also computed:

$$r = \frac{(\mu_n - \mu^*)^2 + (\nu_n - \nu^*)^2}{c + (\mu_n + \mu^*)^2 + (\nu_n + \nu^*)^2} \quad . \tag{78}$$

Using a value for c of 0.4, the report is rejected if $r > 0.5$. This test is most severe in light wind speed cases and causes rejection of many light or calm nighttime wind reports which occur as a result of high

boundary layer stability. High elevation and valley reports, which are also nonrepresentative of the sea-level pressure gradient, are rejected by this test as well.

The relative merit of the separate first-guess fields and their derivatives can also be evaluated. In the case of the first-guess pressure, the ratio

$$F = \frac{\left(p_e - p^*\right)^2}{\left(p_e - p^*\right)^2 + \left(p_x - p^*\right)^2} \tag{79}$$

is computed where p_e is the prognostic field and p_x is the extrapolated field. The separate first guesses can now be reweighted to form the new first guess:

$$p_{oR} = Fp_x + (1 - F)\, p_e \quad . \tag{80}$$

The first-guess reliability, A_o, is not changed. The same procedure is applied to the first-guess difference and Laplacian fields. The new first-guess fields and the reevaluated data are saved for the reanalysis cycle.

3.6 Final Analysis

The purpose of the first analysis cycle is only to get an accurate evaluation of all data and first-guess fields for use in the final analysis.

The reanalysis begins by returning to the assembly stage of the program. The new assembly is initialized with the reweighted first-guess fields and the reports are assembled with their reevaluated weights. Gross error and lateral rejects are omitted.

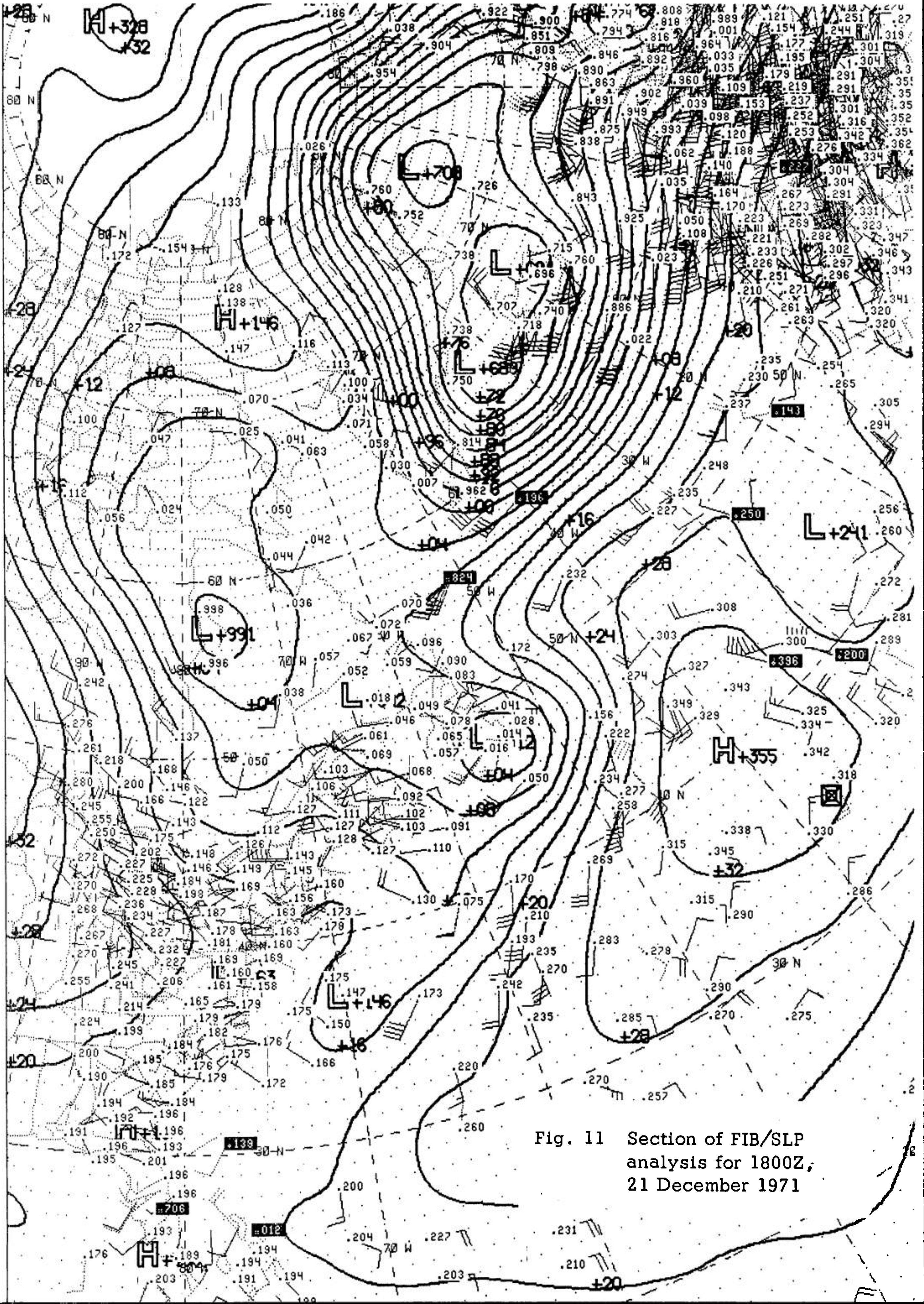

Fig. 11 Section of FIB/SLP analysis for 1800Z, 21 December 1971

The blending for p^* follows exactly the same procedure described in Section 3.3 except that, to aid convergence, the iteration is initialized with the p^* field computed in the first cycle. The A^* values are not needed by the program in the final analysis but may be computed for product use. The program skips the reevaluation stage and proceeds to the output section. The final analysis is stored on the disc in 125x125, 89x89 and 63x63 grid-point form. The smaller arrays are used as input to other FNWC programs. The full resolution analysis is saved for future first-guess use by FIB/SLP.

4. Sample Analysis and Evaluation

A section of a FIB/SLP analysis is shown in Fig. 11. Further evaluation of FIB/SLP should consider the character shown in the analyzed fields, discrimination in rejecting reports, and use of first-guess information. A complete evaluation of FIB/SLP will be possible only after a suitable period of operational testing and use.

Appendix

FIB/SLP Adjustable Constants

Symbol	Value	Purpose	
f_o	$16\ mb^2$	computation of D_o	(constant)
f_1	1.2	computation of D_o	($\overline{L^2}$ term)
f_2	4 mb	pressure gross error check	(constant)
f_3	2.8	pressure gross error check	($\lvert\overline{g}\rvert$ term)
f_4	20 mb	pressure gross error check	(1 - cos φ term)
f_5	$0.5\ mb^2$	computation of A_n	(constant--minimum report variance)
f_6	0,003	computation of A_n	($\overline{g^2}$ term)
f_7	$1x10^{-6} mb^2 m^{-2}$	computation of A_n	(E^2 term)
f_8	$0.1\ mb^2 hr^{-2}$	computation of A_n	(H^2--constant term)
f_9	$0.02\ hr^{-2}$	computation of A_n	($H^2\overline{g^2}$ term)
f_{10}	$1.0\ mb^2$	computation of B_n , C_n	(constant--min.variance)
λ^2_{maxp}	15	lateral reject limit, pressure	
λ^2_{maxw}	8	lateral reject limit, wind	

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13. ABSTRACT

A comprehensive technique for the objective analysis if scalar and of vector fields has been developed by Meteorology International Incorporated under Navy contracts. This technique has been designated the Fields by Information Blending (FIB) technique. In the FIB context, all information statements must include both parameter estimates and associated reliabilities. For an independant piece of information the reliability, or report weight, is defined as the inverse of the error variance inherent in the observation and/or associated with the class of observation.

This report discusses the application of this powerful technique to the analysis of sea level pressure.

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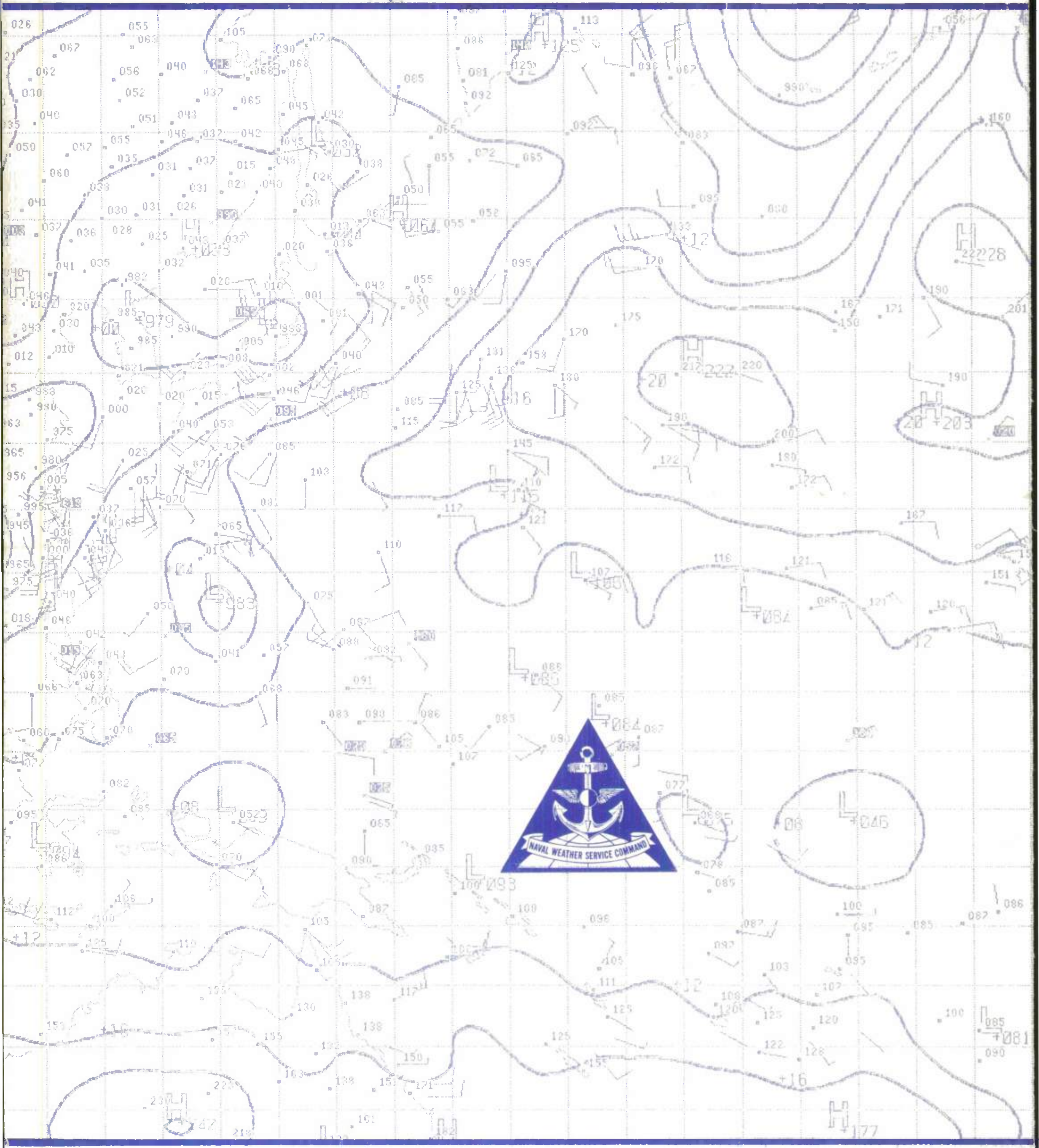
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